

Thermodynamique

Changements d'états

Exercice 1

eau à 100°C	h	u	s	v
V	2676	~~1,6789~~ 2506,5	7,35	1,6789
L	419	418,94		

$$1) \cdot h_{\text{vap}}(100^\circ) = h_V - h_L \\ = 2257 \text{ kJ/kg}$$

$$\cdot s_{1 \rightarrow 2}(T_0) = \frac{h_{1 \rightarrow 2}(T_0)}{T_0} \quad \text{ici} \quad s_{\text{vap}}(100^\circ\text{C}) = \frac{h_{\text{vap}}(100^\circ\text{C})}{373,15} \\ = 6,05 \text{ kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$$

$$\text{on} \quad s_{\text{vap}} = s_V - s_L \\ s_L = 1,30 \text{ kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$$

$$2) u_{\text{vap}} = u_V - u_L = 2087,56 \text{ kJ} \cdot \text{kg}^{-1}$$

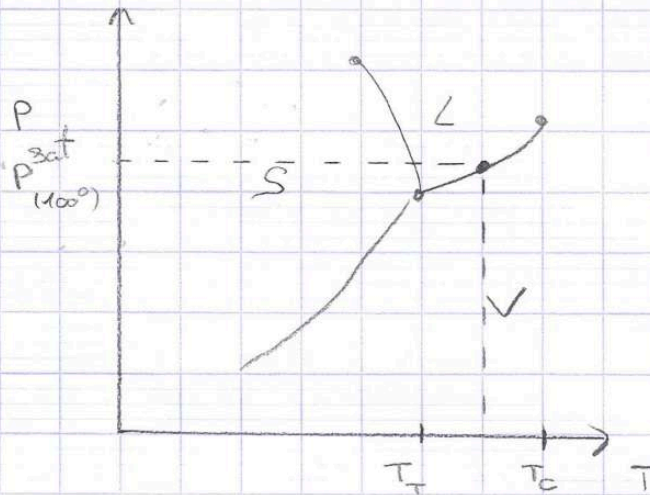
$$h_{\text{vap}} = u_{\text{vap}} + p^{\text{sat}}(T)(v_V - v_L)$$

$$v_L = v_V - \frac{h_{\text{vap}} - u_{\text{vap}}}{p^{\text{sat}}(T)} = 0,0011 \text{ m}^3/\text{kg}$$

$$\text{on} \quad \rho_L = \frac{1}{v_L} = 909 \text{ kg/m}^3 \Rightarrow \text{Faux car les mesures}$$

$$\text{moment que } \rho_L(100^\circ, 1 \text{ atm}) = 961 \text{ kg} \cdot \text{m}^{-3}$$

3)



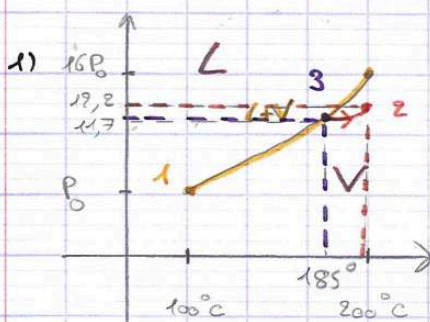
$$4) \left(\frac{\partial P}{\partial T} \right)_{T_0} = \frac{h_{\text{vap}}(T_0)}{T_0 (v_v - v_L)} = \frac{2257 \cdot 10^3}{393 (1,6729 - 0,0011)} = 3620 \text{ Pa/K}$$

~~Met~~ ~~100~~ ~~°C~~ ~~à~~ ~~100~~ ~~°C~~ eau celsius

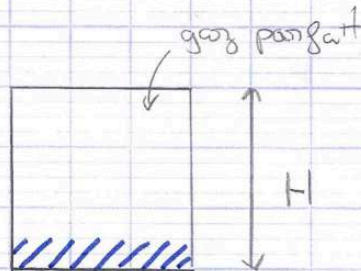
$$5) \left(\frac{\partial P}{\partial T} \right)_{0^\circ\text{C}} = \frac{h_{\text{gl}}(0^\circ\text{C})}{T(v_L - v_S)} = -1,35 \cdot 10^3 \text{ Pa/K}$$

$$\approx -135 \text{ bar/K}$$

Exercice 3 : Eau liquide et eau vapeur dans une chaudière



2)



$$PV = m_v R (\theta_1 + 273)$$

$$V = (H - h)S$$

$$V \sim HS \text{ car } h \ll H$$

équation liq, vap, $\theta = \theta_1 = 100^\circ\text{C}$

$$\Rightarrow P = P^s(\theta_1) = P_0$$

$$m_{v1} = \frac{P_0 SH}{R(\theta_1 + 273)}$$

$$= 32,1 \text{ mol}$$

3) $m_{p1} = ?$

$$M = 18 \text{ g} \cdot \text{mol}^{-1}$$

$$\rho = 1 \text{ g} \cdot \text{cm}^{-3}$$

$$m = \rho V = \rho Sh$$

$$m_{p1} = \frac{m}{M} = \frac{\rho Sh}{M}$$

$$m_{p1} = 277,8 \text{ mol}$$

$$\begin{aligned} 4) \quad m_{v1} &= \frac{P^s(\theta_1) SH}{R(\theta_1 + 273)} \\ &= \frac{\left(\frac{\theta_1}{100}\right)^4}{(\theta_1 + 273)} \cdot \frac{P_0 SH}{R} \end{aligned}$$

$$\begin{aligned} m_v &\text{ devient donc} \\ m_v &\approx \frac{P_0 SH}{R} \cdot \frac{(\theta/100)^4}{\theta + 273} \end{aligned}$$

5) état 2, $\theta_2 = 200^\circ\text{C}$, $m_{liq2} = 0$

$$\Rightarrow m_{vap2} = m_{tot} = m_{vap1} + m_{p1}$$

$$P_2 V = m_{tot} R(\theta_2 + 273)$$

$$\Rightarrow P_2 = \frac{(m_{vap1} + m_{p1}) R(\theta_2 + 273)}{SH}$$

$$= 12,2 \text{ bars}$$

6) Dernière goutte de liquide ③

$$m_{liq3} = 0 \quad \text{mais} \quad P_3 = P^{sat}(\theta_3)$$

$$m_{vap3} = 0$$

$$\Rightarrow \text{que du gaz} \Rightarrow P_3 V = m_{tot} R(\theta_3 + 273)$$

$$P_0 \left(\frac{\theta_3}{100}\right)^4 SH = (m_{p1} + m_{vap1}) R(\theta_3 + 273)$$

On a donc un polynôme de la forme $a\theta^4 + b\theta + c = 0$

$$m_{\text{tot}} = m_v = \frac{P_0 S H}{R} \frac{(\theta/100)^4}{\theta + 273}$$

$$\Rightarrow \theta_3 = 185^\circ \text{C}$$

$$P^s(\theta_3) = 11,7 \text{ bars}$$

Exercice 4: détente d'une vapeur saturante sèche

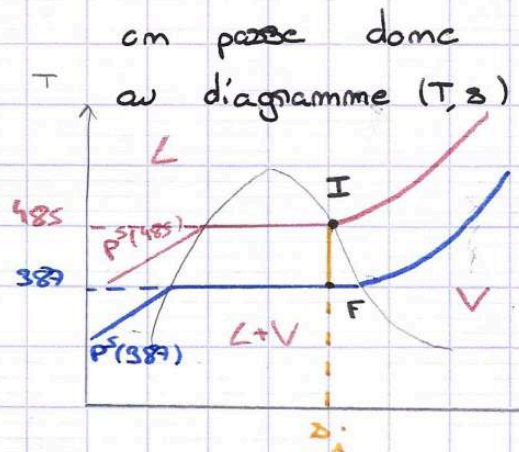
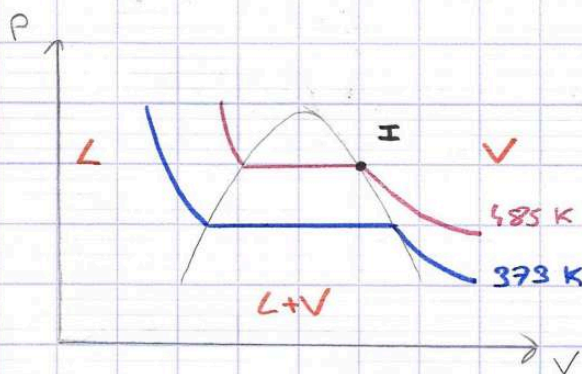
détente : $V \rightarrow$

$$T_1 \rightarrow T_2$$

$$185 \text{ K} \rightarrow 373 \text{ K}$$

vapeur saturante sèche $\rightarrow x=1$

$\hookrightarrow$ limite du liquide



$$v_i = \frac{V_i}{m}$$

La transformation est une isentropie

$$v_g = \frac{V_g}{m}$$

$$\Rightarrow s = \text{cte}$$

$x_i = 1$ on a 100% de vapeur

$$x = \frac{LM}{LV} = \frac{s(M) - s(L)}{s(V) - s(L)} = 0,83 \quad \text{donc } 83\% \text{ de vapeur}$$

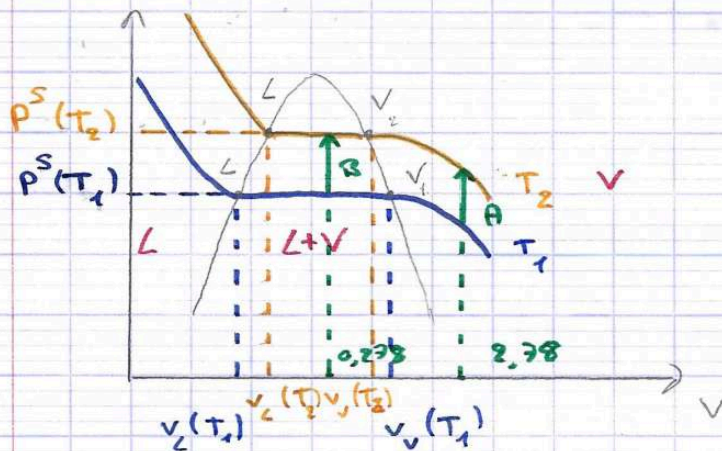
$$x = \frac{LM}{LV} = \frac{v(M) - v_L}{v_V - v_L}$$

$$v(M) = x v_V + (1-x) v_L$$

$$\Rightarrow v_g = 1,416 \text{ m}^3 \cdot \text{kg}^{-1}$$

Exercice 6: Chauffage d'une marmite

1)



2)

$$v_A = \frac{V_A}{m_A} = 2,78 \text{ m}^3 \cdot \text{kg}^{-1} \quad x_A = 1 \quad \text{vapeur}$$

$$v_B = \frac{V_B}{m_B} = 0,278 \text{ m}^3 \cdot \text{kg}^{-1} \quad x_B = \frac{L_B}{L_V} = 16 \%$$

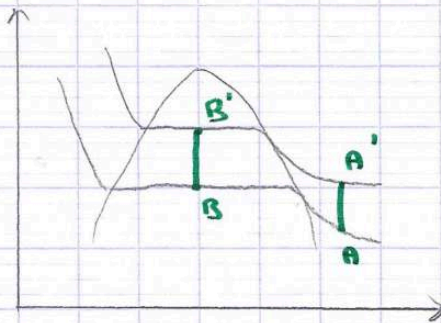
Transformation $T_A \rightarrow T_B$

3) a)

$v = \text{cte}$ schéma $\Rightarrow$ A' vapeur
B' l. q + vap

$$x_B' = \frac{v - v_L(T_2)}{v_V(T_2) - v_L(T_2)} = 0,71\%$$

b)



c) On cherche Q :

$$Q = \Delta U$$

$$= \Delta U_A + \Delta U_B$$

$$\begin{aligned} \bullet \underline{A \rightarrow A'} : \text{vapeur} \rightarrow \text{g.p} &\rightarrow \Delta U_{A'A} = C_v (T_2 - T_1) \\ &= m_A C_v (T_2 - T_1) \end{aligned}$$

$$\begin{aligned} \bullet \underline{B \rightarrow B'} \quad \Delta U_{B'B} &= \Delta U_{BV_1} + \Delta U_{V_1V_2} + \Delta U_{V_2B'} \\ &\quad \parallel \\ &\quad m_B C_v (T_2 - T_1) \end{aligned}$$

$$\begin{aligned} \Delta U_{BV_1} &= m_B \Delta U_{BV_1} \\ &= m_B \int_B^{V_1} du \end{aligned}$$

$$dh = du + p^{sat} dv$$

$$\Rightarrow du = dh - p^{sat}(T) dv$$

$$\Rightarrow \Delta U_{BV_1} = \int_B^{V_1} dh - \int_B^{V_1} p^{sat}(T_1) dv$$

$$\begin{aligned} \text{on } dh &= h_{1 \rightarrow 2}(T) dx_2 \\ &= h_{vap}(T) dx \end{aligned}$$

$$\Delta U_{BV_1} = \int_{x_B}^1 h_{vap}(T_1) dx - p^{sat}(T_1) \int_B^{V_1} dv$$

$$\Delta U_{BV_1} = h_{vap}(T_1) [1 - x_B] - p^{sat}(T_1) [v_v(T_1) - v_B]$$

$$\Delta U_{V_2B'} = h_{vap}(T_2) [x_{B'} - 1] - p^{sat}(T_2) [v(B') - v_v(T_2)]$$

donc :

$$\begin{aligned} \Delta U &= m_A C_v (T_2 - T_1) + m_B C_v (T_2 - T_1) + m_B h_{vap}(T_1) [1 - x_B] \\ &\quad + m_B h_{vap}(T_1) [x_{B'} - 1] - m_B p^{sat}(T_1) [v_v(T_1) - v_B] \\ &\quad - m_B p^{sat}(T_2) [v(B') - v_v(T_2)] \end{aligned}$$

$$d) \quad \Delta S = \Delta S_{A \rightarrow A'} + \Delta S_{B \rightarrow B'}$$

$$du = T ds - P dv$$

$$ds = \frac{1}{T} du + \frac{P}{T} dv$$

$$\text{on passe par } \begin{cases} du = c_v dT \\ \frac{P}{T} = \frac{nR}{V} \end{cases}$$

$$\Rightarrow ds = c_v \frac{dT}{T} + nR \frac{dv}{v}$$

$$\Delta S = \int_A^{A'} ds = \int_{T_1}^{T_2} c_v \frac{dT}{T} + \int_{V_A}^{V_{A'}} nR \frac{dv}{v}$$

$$\Delta S_{AA'} = c_v \ln \left(\frac{T_2}{T_1} \right)$$

$$\Delta S_{AA'} = m_A c_v \ln \left(\frac{T_2}{T_1} \right)$$

B → B' en passant par L_1 et L_2

$$\Delta S_{BB'} = \Delta S_{BL_1} + \Delta S_{L_1 L_2} + \Delta S_{L_2 B'}$$

$$\Delta S_{L_1 L_2} = g_{ac} \cdot P_e$$

$$dh = c dt$$

$$ds = \frac{dh}{T} = c \frac{dT}{T}$$

$$\Delta S_{L_1 L_2} = m_B \cdot c \cdot \ln(T_2/T_1)$$

$$ds_{1 \rightarrow 2} = \frac{dh_{1 \rightarrow 2}}{T_0} \quad \text{donc} \quad \Delta S_{BL_1} = \int_B^{L_1} \frac{h_{vap}(T_1)}{T_1} dx$$

$$\Rightarrow \Delta S_{BL_1} = \frac{h_{vap}(T_1)}{T_1} (-x_B)$$

$$\Delta S_{BL_2} = \frac{h_{vap}(T_2)}{T_2} (x_{B'})$$

Bilan

$$\Delta S = m_A \cdot c_v \ln(T_2/T_1) + m_B c_{rig} \ln(T_2/T_1)$$

$$+ m_B \left[\frac{h_{vap}(T_2) x_{B'}}{T_2} - \frac{h_{vap}(T_1) x_{B}}{T_1} \right]$$

$$= 5535,65 \text{ J.K}^{-1}$$

2nd principe:

$$dS_{\text{source}} = \frac{\delta Q_{\text{source}}}{T_{\text{source}}} = - \frac{\delta Q}{T_{\text{source}}}$$

ici $T_{\text{source}} = T_2$

$$\Rightarrow \Delta S_{\text{source}} = - \int \frac{\delta Q}{T_2} = - \frac{1}{T_2} \int dQ$$

$$\Delta S_{\text{source}} = - \frac{Q}{T_2} = - 5212 \text{ J} \cdot \text{K}^{-1}$$

on 2nd principe $\Delta S = \Delta S_{\text{ech}} + \Delta S_{\text{créé}}$

$$\Rightarrow \Delta S = \frac{Q}{T_2} + S_{\text{créé}}$$

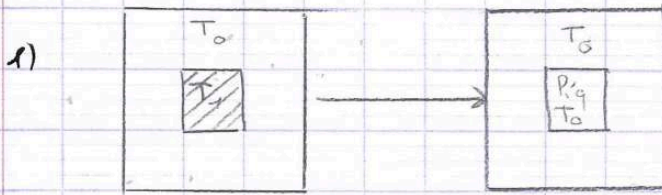
$$\text{Donc } S_{\text{créé}} = \Delta S - \frac{Q}{T_2} = 323 \text{ J} \cdot \text{K}^{-1}$$

cas du système A:

$$S_{\text{ech}} = \frac{Q}{T_2} = \frac{m c_V (T_2 - T_1)}{T_2} = m c_V \left(1 - \frac{T_1}{T_2}\right)$$

$$\text{Et } \Delta S_A = m c_V \ln(T_2/T_1).$$

Exercice 2: Transfert thermique et état métastable



$$\Delta H = \Delta H_1 + \Delta H_2$$

$\Delta H_1 = ?$ corps solide $dh = c dT$

$$\Rightarrow \Delta H = m \Delta h = m \int_{T_1}^{T_0} dh = m \int_{T_1}^{T_0} c dT$$

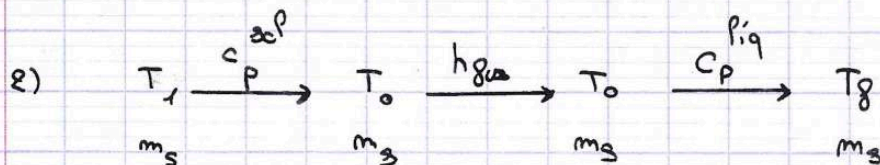
$$\Delta H_1 = m_s c_{p,s} (T_0 - T_1)$$

Changement d'état

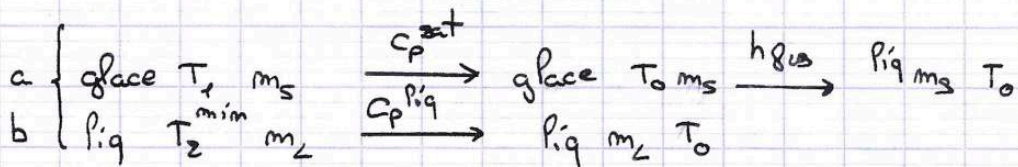
$$\Delta H_2 = m_s h_{\text{fus}} (T_0)$$

$$\Delta H = m_s c_{p,s} (T_0 - T_1) + m_s h_{\text{fus}} (T_0)$$

$$\Delta H = 375,8 \text{ kJ}$$



ici T_g min tel que $T_g = T_0$ et glaçon fondue



$$\Delta H_a = m_s c_{p,s} (T_0 - T_1) + m_s h_{\text{fus}}$$

$$\Delta H_b = m_L c_p^{\text{liq}} (T_0 - T_2^{\text{min}})$$

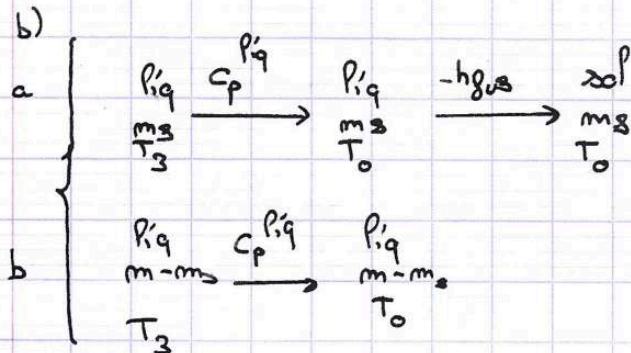
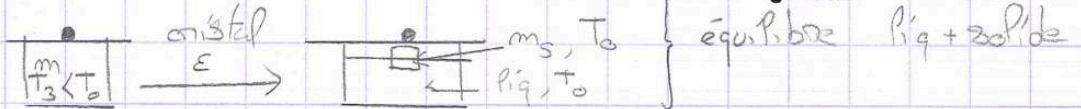
ici calorimètre isolé et capacité calorifique = 0

$$\Delta H_{\text{tot}} = 0$$

$$\Rightarrow \Delta H_a + \Delta H_b = 0 \Rightarrow T_2^{mim} = T_0 + \frac{m_s c_p^{sol} (T_0 - T_1) + m_s h_{fus}}{m_c c_p^{liq}}$$

3) m

a) $T_3 < T_0$



$$\Delta H = \Delta H_a + \Delta H_b$$

$$\Delta H_a = c_p^{liq} m_s (T_0 - T_3) - m_s h_{fus}$$

$$\Delta H_b = (m - m_s) c_p^{liq} (T_0 - T_3)$$

$$\text{on } \Delta H = 0$$

$$\Rightarrow m c_p^{liq} (T_0 - T_3) - m_s h_{fus} = 0$$

$$m_s = \frac{m c_p^{liq} (T_0 - T_3)}{h_{fus}}$$

$$c) \Delta S^{recple} = S_{ech}^{recple} + S_{encc}^{recple}$$

j'immagine une transformation réversible ayant les mêmes états i et f.

$$\Rightarrow \Delta S^{img} = S_{ech}^{img} + \cancel{S_{encc}^{img}} = \Delta S^{recple}$$

$\Rightarrow$ $\Delta S_{\text{réelle}} = \Delta S_{\text{calcule}} = \Delta S_{\text{ech}}^{\text{img}}$
 $\Rightarrow \Delta S = \Delta S_e^{\text{img}} + \Delta S_b^{\text{img}}$

$\Delta S_b^{\text{img}} = ?$ modèle liquide $ds = c \frac{dT}{T}$

$\Rightarrow \Delta S_b = (m - m_s) \int_{T_3}^{T_0} c \frac{dT}{T}$

$\Delta S_b = (m - m_s) c \ln \left(\frac{T_0}{T_3} \right)$

$\Delta S_a^{\text{img}} = ?$

partie 1 pour liq $T_3 \rightarrow T_0$

$\Delta S_a^1 = m_s c \ln \left(\frac{T_0}{T_3} \right)$

partie 2 "chgmt d'état"

$ds_{f \rightarrow s} = \frac{dh_{f \rightarrow s}}{T_0}$

$\Delta s_{f \rightarrow s} = \frac{\Delta h_{f \rightarrow s}}{T_0}$

$\Rightarrow \Delta S_a = \frac{\Delta h_{f \rightarrow s}}{T_0} = \frac{-m_s h_{fs}}{T_0}$

Bilan

$\Delta S^{\text{img}} = (m - m_s) c \ln \frac{T_0}{T_3} + m_s c \ln \frac{T_0}{T_3} - \frac{m_s h_{fs}}{T_0}$

$\Delta S^{\text{img}} = m c_p^{\text{liq}} \ln \left(\frac{T_0}{T_3} \right) - \frac{m_s h_{fs}}{T_0}$

d) $m_s = 0,125 \text{ kg}$

$\Delta S^{\text{neel}} = \Delta S^{\text{img}}_{\text{neel}} = 3,05 \text{ J.K}^{-1}$

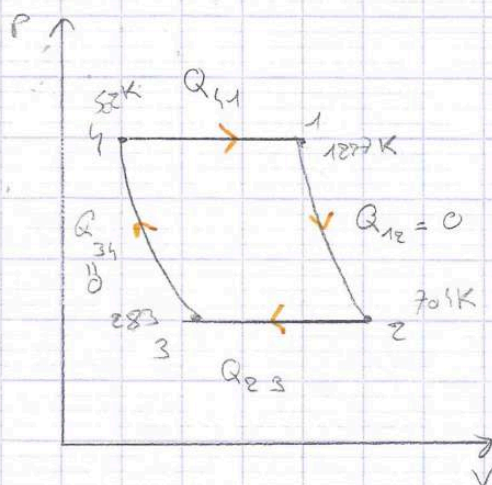
$= S_{\text{ech}}^{\text{neel}} + S_{\text{créé}}^{\text{neel}}$

$\Rightarrow S_{\text{créé}}^{\text{neel}} = 3,05 \text{ J.K}^{-1}$ donc irréversible

O car
 système
 isolé

Exercice 3

1)

Isentropique $P \propto \frac{1}{V^\gamma}$

	T	P
1	1227	7
2	?	1
3	283	1
4	?	7

$$2) \quad e = -\frac{W}{Q_c}$$

$$\Delta U = W + Q = 0$$

$$\Rightarrow W + Q_c + Q_g = 0$$

$$e = 1 + \frac{Q_g}{Q_c}$$

3) $1 \rightarrow 2$ adiabatique réversible = isentropique

$$\begin{cases} ds = \delta S_{ech} + \delta S_{cnc\acute{e}} \\ ds = \frac{\delta Q}{T} + \delta S_{cnc\acute{e}} \end{cases}$$

$$\text{adiabatique} \Rightarrow \delta Q = 0$$

$$\Rightarrow Q_{12} = 0$$

4) $2 \rightarrow 3$ } réversible + isobare.
 $3 \rightarrow 1$ }

$$dH = Tds + VdP$$

$$\text{isobare } dP = 0$$

$$dH = Tds$$

2^{ème} principe :

$$\begin{aligned} ds &= \delta S_{ech} + \delta S_{cnc\acute{e}} \\ &= \frac{\delta Q}{T} + \delta S_{cnc\acute{e}} \end{aligned}$$

$$\Rightarrow Tds = \delta Q$$

$$s) dH = \delta Q \quad \text{sur} \quad 2 \rightarrow 3 \quad \Rightarrow \quad \int_2^3 \delta Q = \int_2^3 dH$$

$$1 \rightarrow 1$$

$$Q_{23} = \Delta H_{23}$$

gaz parfait $\Rightarrow \Delta H_{23} = C_p (T_3 - T_2)$

$$Q_{23} = C_p (T_3 - T_2)$$

idem $Q_{14} = C_p (T_4 - T_1) > 0$

6) $1 \rightarrow 2$ Loi de Laplace
 $3 \rightarrow 4$

$$T^\gamma P^{1-\gamma} = \text{cste}$$

$$\frac{T_1^\gamma P_1^{1-\gamma}}{T_2^\gamma P_2^{1-\gamma}} = 1$$

$$\Rightarrow T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

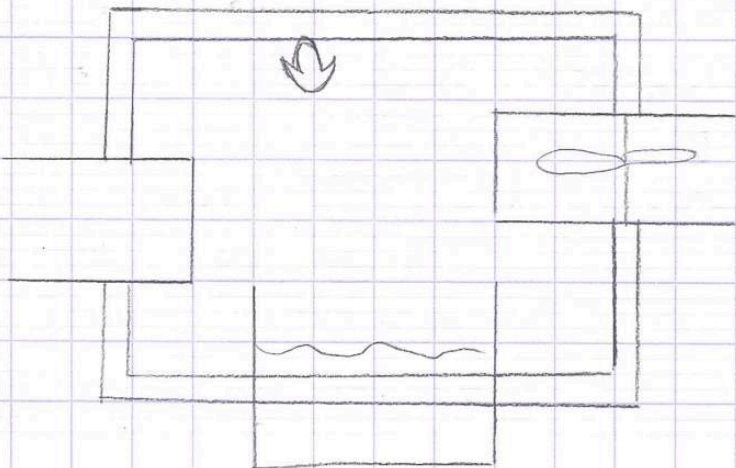
$$T_2 = 1227 \left(\frac{1}{7} \right)^{\frac{0.4}{1.4}} = 704 \text{ K}$$

idem $T_4 = T_3 \left(\frac{P_4}{P_3} \right)^{\frac{\gamma-1}{\gamma}} \approx 502 \text{ K}$

7) $Q_c > 0$ donc $Q_c = Q_{14} (T_4 > T_1)$
 $Q_g < 0$ donc $Q_g = Q_{23} (T_3 < T_2)$

$$\text{on } e = 1 + \frac{Q_g}{Q_c} = 1 + \frac{C_p (T_3 - T_2)}{C_p (T_4 - T_1)} \Rightarrow e = 1 - \frac{T_2 - T_3}{T_4 - T_1}$$

8)



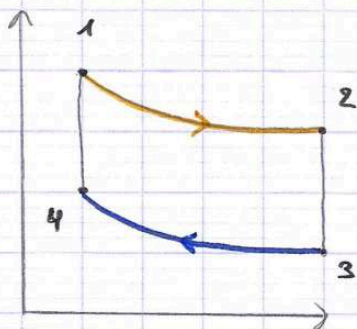
Exercice 10

1) Imcompos

 P_1, P_2

$$PV = nRT$$

$$\Rightarrow n = 3,76 \cdot 10^{-2} \text{ mol}$$



$$W_{12} = -1,08 \text{ kJ} = -nRT_c \ln(V_2/V_1)$$

$$Q_{12} = 1,08 \text{ kJ} = -W_{12}$$

$$Q_c > 0$$

$$W_{23} = 0$$

$$Q_{23} = \Delta U_{23} = \frac{nR}{\gamma-1} (T_f - T_c) = -0,922 \text{ kJ}$$

$$W_{34} = nRT_f \ln(V_4/V_3) = 0,23 \text{ kJ}$$

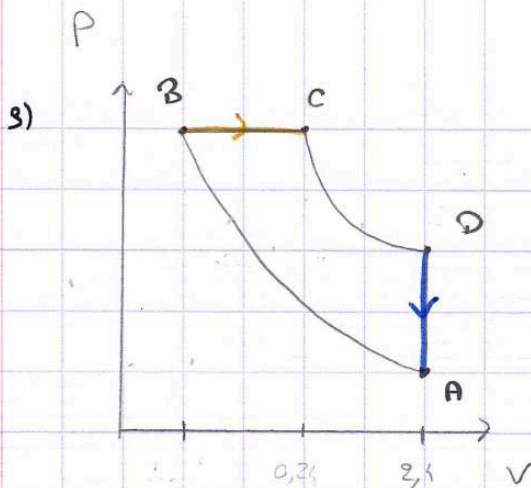
$$Q_{34} = -W_{34} = -0,23 \text{ kJ}$$

$$e = \frac{-W}{Q_c} = 1 + \frac{Q_f}{Q_c} = 0,79 = e_c \quad \text{à cause de négé-}$$

menation.

Exercice 11

1)		A	B	C	D
ben P		1	44,3	4,3	1,76
L V		2,1	0,16	0,24	2,1
K T		323	954	1431	570



moteur 2

$$TV^{\gamma-1} = \text{cte}$$

$$V_B = V_A \left(\frac{T_A}{T_B} \right)^{\frac{1}{\gamma-1}}$$

$$= 0,16 \text{ L}$$

$$PV = nRT \Rightarrow P_B = 44,3 \text{ bar}$$

$$PV = nRT \Rightarrow T_C = 1131 \text{ K}$$

$$TV^{\gamma-1} = \text{cte} \text{ sur CD}$$

$$\Rightarrow T_D = 570 \text{ K}$$

$$P_D = 1,76 \text{ bar}$$

$$2) C_V = \frac{nR}{\gamma-1} = 1,86 \text{ J.K}^{-1}$$

$$C_P = \gamma C_V = 2,6 \text{ J.K}^{-1}$$

1) A $\rightarrow$ B sont fermées

$$\Delta U_{AB} = W_{AB} + Q_{AB}$$

$\rightarrow$ adiabatique réversible $Q_{AB} = 0$

$$\text{donc } W_{AB} = \Delta U_{AB}$$

$$\text{on } \Delta U_{AB} = C_V (T_B - T_A) \text{ car gaz parfait}$$

$$W_{AB} = C_V (T_B - T_A) = 170 \text{ J}$$

$$3) \Delta U_{BC} = C_V (T_C - T_B) = W_{BC} + Q_{BC}$$

$$\Delta H_{BC} = C_P (T_C - T_B) = Q_{BC}$$

$$dU = Tds - PdV = \delta W + \delta Q$$

$$dH = Tds + VdP$$

$$dH = \delta W + \delta Q + PdV + VdP$$

$$Q_{BC} = C_p (T_c - T_B)$$

$$W_{BC} = C_v (T_c - T_B) - C_p (T_c - T_B)$$

$$6) \begin{cases} W_{CO} = ? \text{ idem} \\ Q_{CO} = ? \\ Q_{CD} = 0 \\ W_{CD} = C_v (T_D - T_c) \end{cases}$$

$$7) \text{ isochore } \begin{cases} W_{DA} = 0 \\ \Delta U_{DA} = Q_{DA} \end{cases} \quad \text{donc } Q_{DA} = C_v (T_A - T_D)$$

$$8) \begin{aligned} W_{ABCD} &= C_v (T_B - T_A) + C_v (T_c - T_B) - C_p (T_c - T_B) + C_v (T_D - T_c) + 0 \\ &= C_v (T_D - T_A) - C_p (T_c - T_B) \\ &= -780 \text{ J} \end{aligned}$$

$$9) \quad e = \frac{u}{d} = \frac{-W}{Q_c} = \frac{-W_{ABCD}}{Q_{BC}}$$

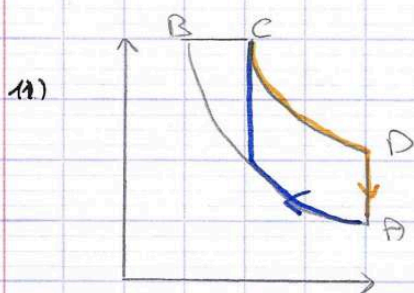
$$= 1 - \frac{C_v}{C_p} \left(\frac{T_D - T_A}{T_c - T_B} \right)$$

$$= 1 - \frac{1}{\gamma} \frac{T_D - T_A}{T_c - T_B} = 0,63$$

$$10) \quad e_{\text{max}} = e_{\text{Carnot}} = 1 - \frac{T_A}{T_c}$$

$$= 1 - \frac{T_A}{T_c} = 0,77$$

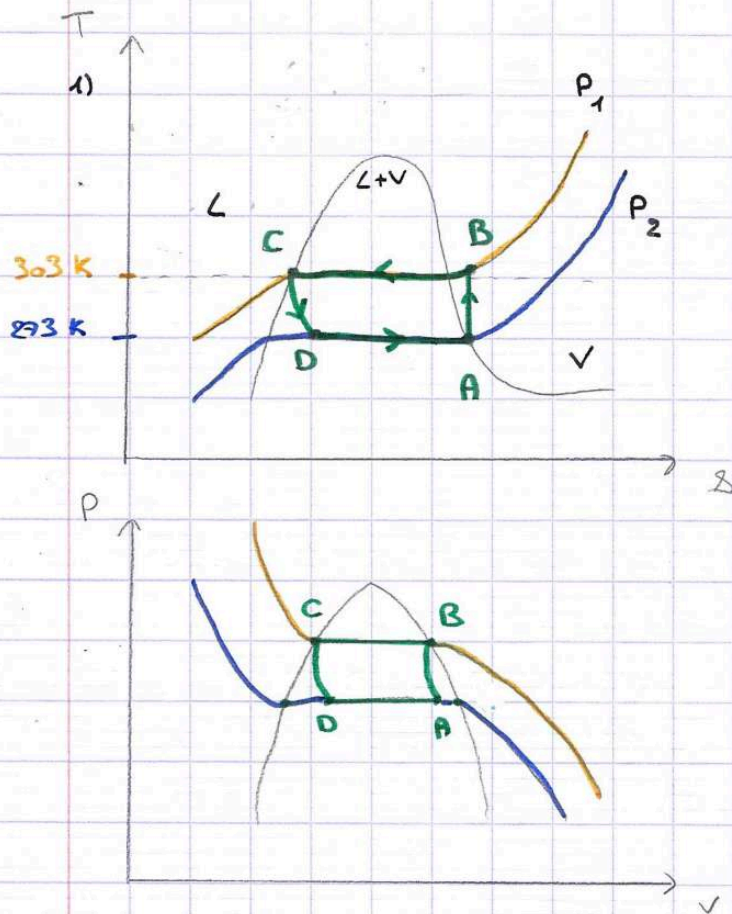
irréversible



$$12) \begin{aligned} W_{AB_1} &= \\ Q_{AB_1} &= \\ \text{if } T_{B_1} &= T_B \\ P_A V_A &= P_{B_1} V_{B_1} \\ \Rightarrow P_{B_1} &= 25,11 \text{ bar} \\ T_{B_1} &= 811 \text{ K} \\ \Rightarrow \begin{cases} W_{AB_1} = C_v (T_{B_1} - T_A) \\ Q_{AB_1} = 0 \end{cases} \\ \Rightarrow \begin{cases} W_{BC} = 0 \\ Q_{BC} = C_v (T_c - T_{B_1}) \end{cases} \end{aligned}$$

$$13) \quad e_{BDR} = 1 - \frac{T_D - T_A}{T_c - T_{B_1}} = 0,6$$

Exercice 12



2)

$$\begin{cases} h_C = h_L(303\text{ K}) \\ h_D = x_D h_V(273\text{ K}) + (1-x_D) h_L(273\text{ K}) \end{cases}$$

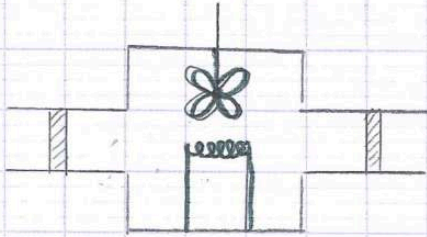
Transfo imaginaire

$$\begin{aligned} \Delta h_{CD} &= \Delta h_{CC'} + \Delta h_{C'D} = 0 \\ &= c(T_D - T_C) + h_{\text{vap}}(273\text{ K})(x_D - 0) = 0 \end{aligned}$$

$$x_D = \frac{c(T_C - T_D)}{h_{\text{vap}}(273\text{ K})} = 9,7\%$$

3) 1^{er} principe adapté au système ouvert

$$\Delta h = w_{\text{autres}} + q$$



Compresseur calorigène $\Rightarrow q = 0$

donc $w_{cp} = w_{autres} = \Delta h_{AB} = h_B - h_A$

gaz parfait $\Rightarrow \Delta h_{AB} = c_p (T_B - T_A)$

$\Rightarrow w_{cp} = c_p (T_B - T_A)$

$\Rightarrow W = m w_{cp} = m c_p (T_B - T_A)$

1) $e = \frac{u}{d} = \frac{q_D}{w} = \frac{q_{DA}}{w_{cp}}$

$e = \frac{h_{vap}(273K)(1-x_D)}{c_p (T_B - T_A)}$

$q_{DA} = h_{vap}(273) [1 - x_D]$

$w_{cp} = c_p (T_B - T_A)$

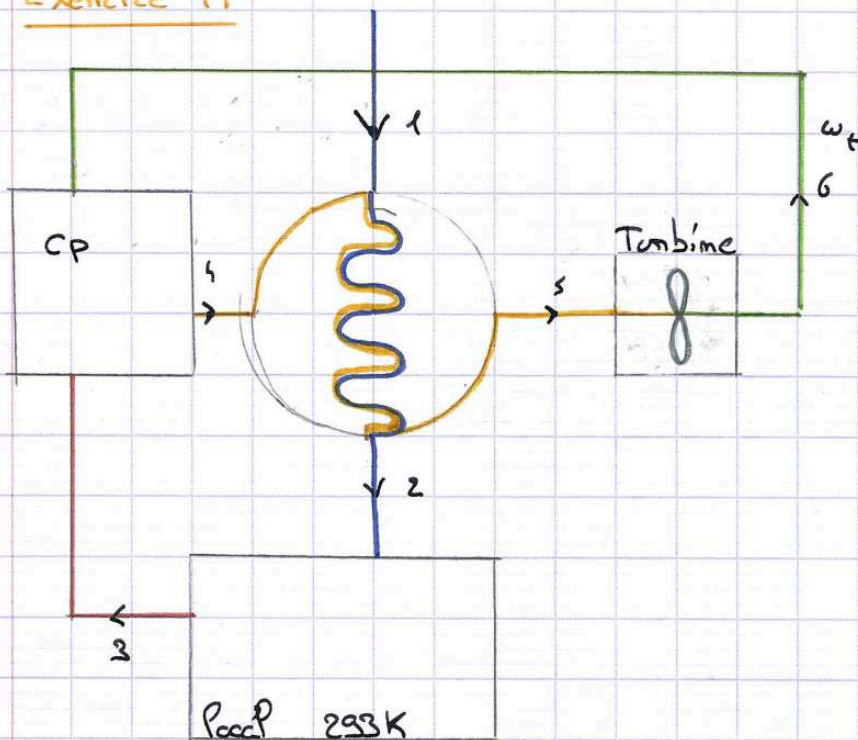
A $\rightarrow$ B isentropique

$T_B = T_A \left(\frac{P_B}{P_A} \right)^{\frac{1-\gamma}{\gamma}} = T_A \left(\frac{P_B}{P_A} \right)^{\frac{\gamma-1}{\gamma}}$ $PV^\gamma = \text{cte} \Rightarrow P^{1-\gamma} T^\gamma = \text{cte}$

$e = \frac{h_{vap}(273K)(1-x_D)}{c_p T_A \left[\left(\frac{P_B}{P_A} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]} = 7,5$

$e_c = \frac{T_F}{T_C - T_F} = 18,5$

Exercice 11



1) 1 bar
-5°C

2) $T_2 = 351 \text{ K}$
 $P_2 = 1 \text{ bar}$

3)

4) 2 bars
135 K

5) 2 bars
351 K

6) 1 bar
236 K

1) $3 \rightarrow 4$ } isos $\rightarrow T^\gamma P^{1/\gamma} = \text{cte}$
 $5 \rightarrow 6$ }

2) $w_{34} = w_{cp} = \frac{\gamma R}{(\gamma-1)M} (T_4 - T_3)$

3) $w_{56} = \frac{\gamma R}{(\gamma-1)M} (T_6 - T_5)$

	1	2	3	4	5	6
P	1	1	1	4	4	1
T	268	351	253	435	351	236

$$2) w_{34} = \frac{\gamma R}{(\gamma-1)M} (T_4 - T_3) = 142,5 \text{ kJ.kg}^{-1}$$

$$3) w_{56} = \frac{\gamma R}{(\gamma-1)M} (T_6 - T_5) = -115,1 \text{ kJ.kg}^{-1}$$

$$4) w_{tot} = w_{56} - w_{34} = 27,1 \text{ kJ.kg}^{-1}$$

$$5) q_c = q_{32} = \Delta h = \frac{\gamma R}{(\gamma-1)M} (T_3 - T_2) = -58,2 \text{ kJ.kg}^{-1}$$

$$6) e_{PRC} = \frac{w}{q} = \frac{-q_c}{w_{56} + w_{34}} = 2,2$$

$$7) e_c = \frac{T_c}{T_c - T_F} = 11,7$$

Physique de la diffusion

Exercice 16 : Mesure d'une conductivité thermique.

$$1) \vec{j}_{th} = -\lambda \vec{\nabla} T \quad \text{et} \quad \vec{j}_{th}(z) = -\lambda \frac{dT}{dz} \vec{e}_z$$

$$j_{th} = W \cdot m^{-2}$$

$$\lambda = W \cdot m^{-1} \cdot K^{-1}$$

$$2) \varphi_{th} = \iint \vec{j}_{th}(\varphi) \cdot d\vec{S} = \iint j_{th} dS = j_{th}(z) \pi r^2$$

$$3) P = UI = RI^2 \quad \text{donc } R \text{ grand}$$

$$1) \frac{P}{R} = \varphi_{th}$$

$$\frac{U^2}{R} = j_{th}(z) \pi r^2$$

$$j_{th}(z) = \frac{U^2}{R \pi r^2}$$

4) Bilan des flux pendant dt.

$$dU_{in} = j(z, t) dS dt$$

$$dU_{out} = j(z+dz, t) dS dt$$

$$dU = (j(z, t) - j(z+dz, t)) dS dt$$

$$U(t) = u(z, t) dm = u(z, t) dS dz \rho$$

$$U(t+dt) = u(z, t+dt) dS dz \rho$$

$$dU = (\rho u(z, t+dt) - \rho u(z, t)) dS dz$$

$$\frac{\partial u}{\partial t} = -\frac{dj}{dz} \quad \text{stationnaire donc} \quad \frac{\partial j}{\partial z} = 0$$

$$j_{th}(z) = cte$$

$$j_{th} = -\lambda \frac{dT}{dz} = cte$$

$$T(x) = ax + b$$

$$a) \vec{j}_{th}(z) = j_{th} \vec{e}_z$$

$$j_{th} = -\lambda \frac{dT}{dz} = -\lambda \frac{T_L - T_0}{L}$$

$$\vec{j}_{th} = \lambda \frac{T_0 - T_L}{L} \vec{e}_z$$

$$a) T(z) = \frac{-j_{th}}{\lambda} z + T_0$$

$$\text{on } j_{th} = \frac{L^2}{R\pi n^2}$$

$$\text{donc } T(z) = - \frac{L^2}{R\pi n^2 \lambda} z + T_0$$

$$= - \frac{T_L - T_0}{L} z + T_0$$

$$T(L) = - \frac{L^2}{R\pi n^2 \lambda} L + T_0$$

$$T_0 = T_L + \frac{L^2}{R\pi n^2 \lambda} L$$

$$b) U = RI$$

$$\frac{T_0 - T_L}{L} = R_{th} \varphi$$

$$T_0 - T_L$$

$$T(z) = \frac{T_L - T_0}{L} z + T_0$$

$$j = \lambda \frac{T_0 - T_L}{L}$$

$$\varphi = j \cdot S = \frac{\lambda S}{L} (T_0 - T_L)$$

$$T_0 - T_L = \varphi \frac{L}{\lambda S}$$

$$R_{th} = \frac{T_0 - T_L}{\varphi} = \frac{L}{\pi n^2 \lambda} = 11,1 \text{ K} \cdot \text{W}^{-1}$$

$$\pi_{th} = \frac{R_{th}}{L} = \frac{1}{\pi n^2 \lambda} = 27,8 \text{ K} \cdot \text{W}^{-1} \cdot \text{m}^{-1}$$

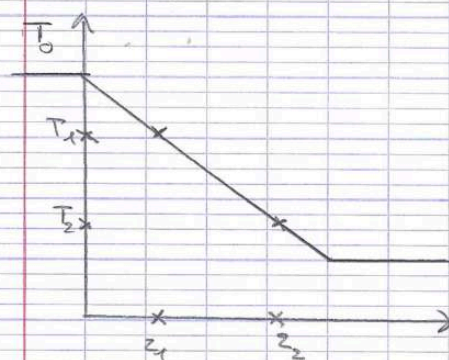
$s) T(z_1) = T_1$
 $T(z_2) = T_2$

$\text{pente} = \frac{T_1 - T_2}{z_1 - z_2}$

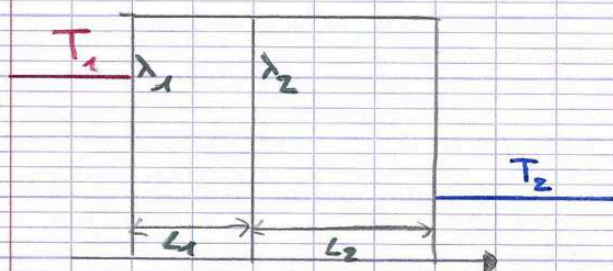
$\frac{1}{\lambda} = \frac{R_{\text{th}}}{\Delta z} = \frac{R_{\text{th}}}{z_1 - z_2} \frac{T_1 - T_2}{T_1 - T_2}$

$j = -\lambda \times \text{pente}$
 or $j = \frac{\Delta z}{R_{\text{th}}}$

$\text{pente} = \frac{\Delta z}{R_{\text{th}}}$



Exercice 17: Diffusion de chaleur à travers une paroi composite



1) Régime stationnaire
 $\rightarrow \Delta T(x, y, z) = 0$
 Problème 1D $\rightarrow T(x)$

$\frac{d^2 T(x)}{dx^2} = 0$

donc $\begin{cases} T_1(x) = a_1 x + b_1 \\ T_2(x) = a_2 x + b_2 \end{cases}$ 4 inconnues.

2) Continuité de T
 Continuité de j en stationnaire

4 conditions aux limites

$T_1(0) = T_1$	1
$T_2(L_1 + L_2) = T_2$	2
$T_1(L_1) = T_2(L_1)$	3
$j_1(L_1) = j_2(L_2)$	

on pose l'anné $\vec{j}_1 = -\lambda_1 \frac{dT_1}{dx} \quad \text{en } x$

$\vec{j}_2 = -\lambda_2 \frac{dT_2}{dx} \quad \text{en } x$

$$\Rightarrow \lambda_1 \frac{dT_1}{dx}(L_1) = \lambda_2 \frac{dT_2}{dx}(L_2) \quad ,$$

$$1 \Rightarrow T_1(0) = b_1 = T_1 \Rightarrow b_1 = T_1$$

$$2 \Rightarrow T_2(L_1 + L_2) = a_2(L_1 + L_2) + b_2 = T_2$$

$$\underline{a_2(L_1 + L_2) + b_2 = T_2}$$

$$\Rightarrow T_1(x) = \frac{dT_1(x)}{dx} x + T_1 = \underline{a_1 x + T_1}$$

$$T_2(x) = \frac{dT_2}{dx} \cdot x + (T_2 - a_2(L_1 + L_2))$$

$$\underline{T_2(x) = \frac{dT_2(x)}{dx} (x - (L_1 + L_2)) + T_2 = a_2(x - (L_1 + L_2)) + T_2}$$

$$1 \Rightarrow \lambda_1 a_1 = \lambda_2 a_2 = \alpha$$

donc $a_1 = \frac{\alpha}{\lambda_1} \quad a_2 = \frac{\alpha}{\lambda_2}$

$$T_1(x) = \frac{\alpha}{\lambda_1} x + T_1$$

$$T_2(x) = \frac{\alpha}{\lambda_2} [x - (L_1 + L_2)] + T_2$$

$$3 \Rightarrow T_1(L_1) = T_2(L_1)$$

$$\Rightarrow \frac{\alpha L_1}{\lambda_1} + T_1 = -\frac{\alpha L_2}{\lambda_2} + T_2$$

$$\Rightarrow \alpha \left(\frac{L_1}{\lambda_1} + \frac{L_2}{\lambda_2} \right) = T_2 - T_1$$

$$\Rightarrow \alpha = \frac{T_2 - T_1}{\frac{L_1}{\lambda_1} + \frac{L_2}{\lambda_2}}$$

donc $T_1(\infty) = \frac{1}{\lambda_1} \left(\frac{T_2 - T_1}{\frac{L_1}{\lambda_1} + \frac{L_2}{\lambda_2}} \right) \infty + T_1$

$$T_2(\infty) = \frac{1}{\lambda_2} \left(\frac{T_2 - T_1}{\frac{L_1}{\lambda_1} + \frac{L_2}{\lambda_2}} \right) (\infty - (L_1 + L_2)) + T_2$$

$$\begin{aligned} 3) T_i &= T_1(L_1) \\ &= \frac{1}{\lambda_1} \frac{T_2 - T_1}{\frac{L_1}{\lambda_1} + \frac{L_2}{\lambda_2}} L_1 + T_1 \end{aligned}$$

$$R = \frac{L}{\lambda S} \quad G = \frac{\lambda S}{L} \quad \begin{cases} G_1 = \frac{\lambda_1}{L_1} S \\ G_2 = \frac{\lambda_2}{L_2} S \end{cases}$$

$$T_i = \frac{L_1}{\lambda_1} \left(\frac{T_2 - T_1}{\frac{L_1 S_1}{\lambda_1 S_2} + \frac{L_2 S_2}{\lambda_2 S_2}} \right) = \frac{L_1}{\lambda_1 S_1} \frac{T_2 - T_1}{\frac{L_1}{\lambda_1 S_2} + \frac{L_2}{\lambda_2 S_2}}$$

$$= \frac{1}{G_1} \frac{T_2 - T_1}{\frac{1}{G_1} + \frac{1}{G_2}} + T_1$$

$$= \frac{(T_2 - T_1) G_2}{G_1 + G_2} + \frac{T_1 (G_1 + G_2)}{G_1 + G_2}$$

$$T_i = \frac{G_1 T_1 + G_2 T_2}{G_1 + G_2}$$

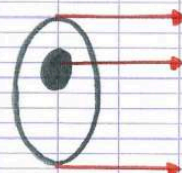
$$4) \vec{j}_1 = -\lambda_1 \frac{dT_1}{dx} \vec{e}_x$$

$$j_1 = -\lambda_1 \left(\frac{1}{\lambda_1} \frac{T_2 - T_1}{\frac{L_1}{\lambda_1} + \frac{L_2}{\lambda_2}} \right)$$

$$j_2 = -\lambda_2 \left(\frac{1}{\lambda_2} \frac{T_2 - T_1}{\frac{L_1}{\lambda_1} + \frac{L_2}{\lambda_2}} \right)$$

$$j = j_1 = j_2 = \frac{T_1 - T_2}{\frac{L_1}{\lambda_1} + \frac{L_2}{\lambda_2}}$$

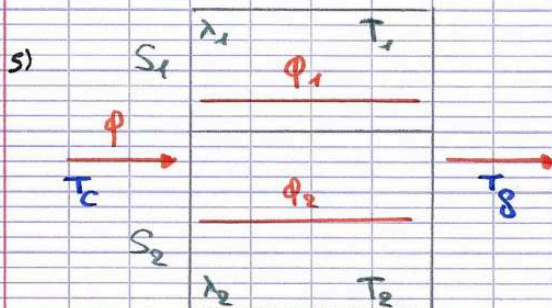
$$\begin{aligned} \varphi &= \iint \vec{j} \cdot d\vec{S} = \iint j \vec{e}_x \cdot d\vec{S} \\ &= j S \end{aligned}$$



$$U = R_{eq} I$$

$$-T_2 + T_1 = \left(\frac{L_1}{\lambda_1 S} + \frac{L_2}{\lambda_2 S} \right) \Phi$$

$$R_{eq} = \frac{L_1}{\lambda_1 S} + \frac{L_2}{\lambda_2 S} = R_1 + R_2$$



$$T_1(x) = a_1 x + b_1$$

$$T_2(x) = a_2 x + b_2$$

$$\begin{cases} T_1(x) = \frac{T_g - T_c}{L} x + T_c \\ T_2(x) = \frac{T_g - T_c}{L} x + T_c \end{cases}$$

$$\vec{j}_1 = -\lambda_1 \frac{dT_1}{dx} \quad \vec{e}_x = \lambda_1 \left(\frac{T_c - T_g}{L} \right) \vec{e}_x$$

$$\vec{j}_2 = -\lambda_2 \frac{dT_2}{dx} \quad \vec{e}_x = \lambda_2 \left(\frac{T_c - T_g}{L} \right) \vec{e}_x$$

$$\Phi_1 = \iint \vec{j}_1 dS = \frac{\lambda_1 S}{L} (T_c - T_g)$$

$$\Phi_2 = \frac{\lambda_2 S}{L} (T_c - T_g)$$

$$\Phi_{tot} = \Phi_1 + \Phi_2$$

$$\Phi_{tot} = \left(\frac{\lambda_1 S}{L} + \frac{\lambda_2 S}{L} \right) (T_c - T_g)$$

$$U = RI \Rightarrow I = \frac{U}{R}$$

$$\Rightarrow \frac{1}{R_{eq}} = \frac{\lambda_1 S}{L} + \frac{\lambda_2 S}{L}$$

$$\Phi_{tot} = \frac{1}{R_{eq}} (T_c - T_g)$$

$$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Transfert conducto-convectif

Cas lorsque l'une des interfaces fait intervenir un fluide (liquide ou gaz)

Solide Fluide

λ_s
 T_s

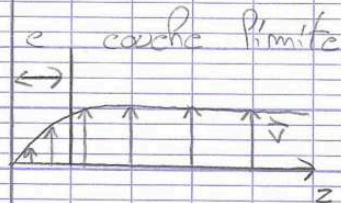
λ_f
 T_f

z

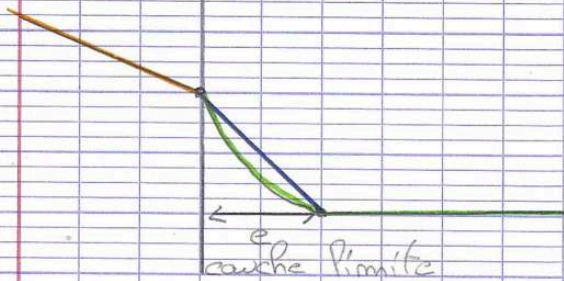
Continuité de $\vec{j}$ à l'interface
 $\vec{j}_s = -\lambda_s \frac{dT_s}{dz} \vec{e}_z$ $\vec{j}_f = -\lambda_f \frac{dT_f}{dz} \vec{e}_z$

$$\lambda_s \frac{dT_s}{dz} = \lambda_f \frac{dT_f}{dz}$$

Solide Fluide



Solide Fluide



$$\vec{j}_f = -\lambda_f \frac{T_f - T_s}{e} \vec{e}_z$$

$$\begin{aligned} \vec{j} &= -\frac{\lambda_f}{e} (T_{\text{fluide}} - T_{\text{paroi}}) \vec{e}_z \\ &= h (T_{\text{paroi}} - T_{\text{fluide}}) \vec{e}_z \end{aligned}$$

$$h = \frac{\lambda_f}{e} \quad \text{coef de transfert conducto-convectif.}$$

T_s λ_s

T_{paroi}

T_{fluide}

T_s

T_{paroi}

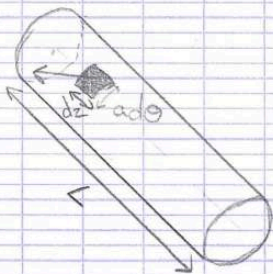
coef h

T_{fluide}

Exercice 18: Isolation d'une canalisation d'eau chaude.

18.1

1)



$$\begin{aligned}\phi &= \iint \vec{j} \cdot d\vec{S} \\ d\vec{S} &= a d\theta dz \vec{e}_r \\ \vec{j} &= h(T_1 - T_0) \vec{e}_r \\ \phi &= \iint h(T_1 - T_0) a d\theta dz \\ \phi &= h(T_1 - T_0) a \int_0^L dz \int_0^{2\pi} d\theta \\ \phi_0 &= h(T_1 - T_0) 2\pi L a\end{aligned}$$

$$2) (T_1 - T_0) = \frac{1}{2\pi L a h} \phi_0$$

$$R_{Th} = \frac{1}{2\pi a L h} = \frac{1}{Sh}$$

18.2

$$1) \Delta T = 0$$

$$T(r, \theta, z)$$

$$\rightarrow T(r)$$

$$2) \Delta g(r) = \frac{1}{r} \frac{d}{dr} \left(r \frac{dg}{dr} \right)$$

$$\Delta T = 0 \Rightarrow \frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) = 0$$

$$\Rightarrow r \frac{dT}{dr} = A$$

$$\frac{dT(r)}{dr} = \frac{A}{r} \Rightarrow T(r) = A \ln r + B$$

2 conditions limites

$$\begin{cases} T(r=a) = T_1 \\ T(r=b) = T_2 \end{cases}$$

$$\begin{cases} T_1 = A \ln a + B \\ T_2 = A \ln b + B \end{cases}$$

$$\begin{aligned} A \ln b - A \ln a &= T_2 - T_1 \\ A &= \frac{T_2 - T_1}{\ln(b/a)} \end{aligned}$$

$$T(r) = \frac{T_2 - T_1}{\rho_m(b/a)} \rho_m(r/a) + B$$

$$T(r) = \frac{T_2 - T_1}{\rho_m(b/a)} \rho_m(r/a) + T_1$$

$$3) \vec{j} = -\lambda \vec{\nabla} T = -\lambda \begin{pmatrix} \frac{\partial T}{\partial r} \\ \frac{1}{r} \frac{\partial T}{\partial \theta} \\ \frac{\partial T}{\partial z} \end{pmatrix}$$

$$T(r) \Rightarrow \vec{j} = -\lambda \frac{dT(r)}{dr} \vec{e}_r$$

$$\frac{dT}{dr} = \frac{T_2 - T_1}{\rho_m(b/a)} = \frac{1}{r}$$

$$\vec{j}(r) = -\lambda \frac{T_2 - T_1}{\rho_m(b/a)} \frac{1}{r} \vec{e}_r$$

$$\vec{j}(a) = -\lambda \frac{T_2 - T_1}{\rho_m(b/a)} \frac{1}{a} \vec{e}_r$$

$$1) \varphi = \iint \vec{j} \cdot d\vec{S}$$

$$\varphi(a) = \iint \vec{j}(a) \cdot d\vec{S}$$

$$= \iint \vec{j}(a) \cdot a d\theta dz \vec{e}_r$$

$$\varphi(a) = \int_0^{2\pi} \int_0^L \lambda \frac{T_1 - T_2}{\rho_m(b/a)} \frac{1}{a} a d\theta dz$$

$$\varphi_1 = \varphi(a) = \lambda \frac{T_1 - T_2}{\rho_m(b/a)} 2\pi L$$

$$5) \varphi_1 = \lambda \frac{T_1 - T_2}{\rho_m(b/a)} 2\pi L$$

$$\varphi_2 = 2\pi L b \cdot h (T_2 - T_0)$$

1) par d'accumulation

de chaleur donc $\varphi_1 = \varphi_2$

$$\varphi_1 = \text{flux en } r=a$$

$$r=b$$

$$2) T_1 - T_2 = \frac{\lambda (T_1 - T_2)}{b h \rho_m(b/a)} T_1 - T_0 + T_0 - T_2$$

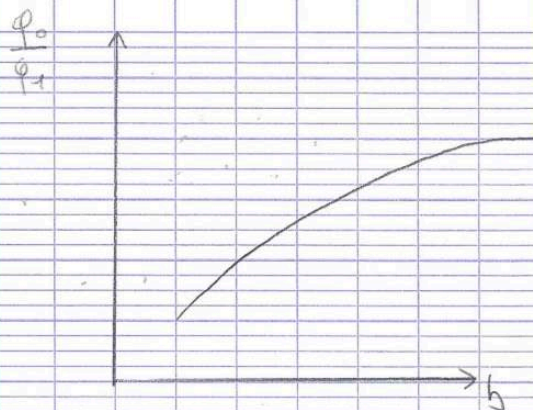
$$T_1 - T_2 = \frac{T_1 - T_0}{1 + \frac{\lambda}{b h \rho_m(b/a)}}$$

$$8) \frac{\varphi_0}{\varphi_1} = \frac{a}{b} + \frac{a h}{\lambda} \rho_m\left(\frac{b}{a}\right)$$

$$9) \text{ si } h=0 \quad \frac{\varphi_0}{\varphi_1} = \frac{a}{b} < 1 \Rightarrow \varphi_0 < \varphi_1$$

$$10) \text{ si } h>0 \quad \frac{b}{a} \text{ grand } b \gg a \quad \frac{a}{b} = \varepsilon$$

$$\frac{\varphi_0}{\varphi_1} = \varepsilon + a \frac{h}{\lambda} \rho_m\left(\frac{1}{\varepsilon}\right)$$



$\frac{q_0}{q_1}$ augmente comme $\ln\left(\frac{b}{a}\right)$ donc ~~est~~ $\ln$ lent
 $\Rightarrow$ peu efficace.

en du coup, on prend $e = b - a$ faible
 $b = a + e$ $\frac{b}{a} = 1 + \frac{e}{a} \rightarrow e$

$$\frac{q_0}{q_1} = \frac{1}{1+e} + \frac{ah}{\lambda} \ln(1+e) \quad \text{if faut } q_0/q_1 > 1$$

$$= 1 - e + \frac{ah}{\lambda} e \quad \Rightarrow \frac{ah}{\lambda} > 1$$

$$= 1 + e\left(\frac{ah}{\lambda} - 1\right) \quad \text{if faut } \lambda < ah$$

$$h > \frac{\lambda}{a}$$

Exercice 19

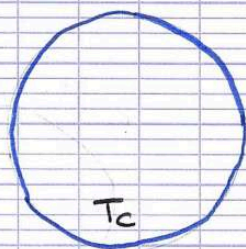
1) $\Delta T = 0$ en sta

2) $T(\eta, \theta, \varphi)$ invariance

de notation $\theta, \varphi \Rightarrow T(\eta) + T = \text{cte}$ con bon
 conduction

$$3) \text{ A } T(\eta) = 0 \Rightarrow \frac{1}{\eta^2} \frac{d}{d\eta} \left(\eta^2 \frac{dT}{d\eta} \right) = 0$$

$$\Rightarrow \eta^2 \frac{dT}{d\eta} = \text{cte} = A'$$



$$\Rightarrow \frac{dT}{dn} = \frac{A'}{n} \Rightarrow T(n) = \frac{A}{n} + B$$

1) Conditions limites

$$T(n=a) = T_c$$

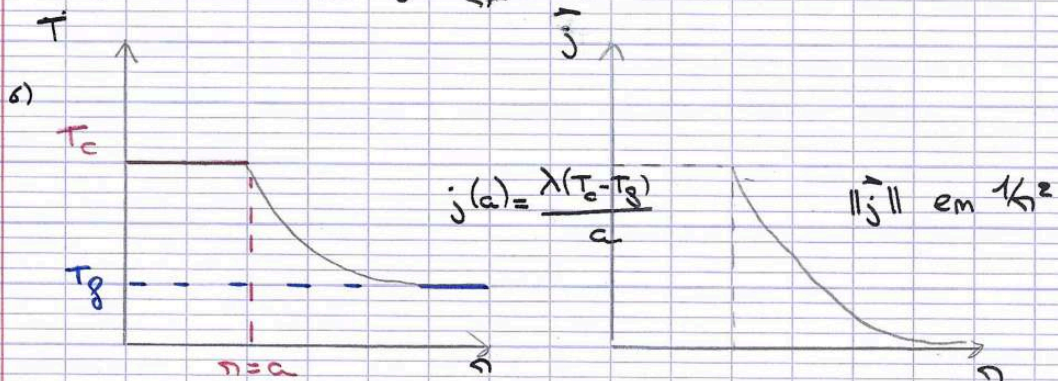
$$T(n \rightarrow +\infty) = T_g$$

$$\begin{cases} A/a + B = T_c \\ 0 + B = T_g \end{cases} \Rightarrow \begin{cases} A = T_c - T_g \\ B = T_g \end{cases}$$

$$T(n) = (T_c - T_g) \frac{a}{n} + T_g$$

$$1) \vec{j} = -\lambda \vec{\nabla} T = -\lambda \frac{dT}{dn} \vec{e}_n = -\lambda (T_c - T_g) \times \left(\frac{-a}{n^2} \right) \vec{e}_n$$

$$\vec{j} = \lambda a (T_c - T_g) \frac{1}{n^2} \vec{e}_n \quad j \text{ em } \frac{1}{n^2}$$



$$\Rightarrow \vec{\phi}_0 = \iint \vec{j} \cdot d\vec{S} \quad \vec{j} = \lambda (T_c - T_g) a \frac{1}{n^2} \vec{e}_n$$

$$dV = n^2 \sin \theta d\theta d\phi$$

$$\phi(n=a) = \iint \frac{\lambda a (T_c - T_g)}{a^2} a^2 \sin \theta d\theta d\phi$$

$$\phi(n=a) = \lambda a (T_c - T_g) \left(\int_0^{2\pi} d\phi \right) \times \left(\int_0^\pi \sin \theta d\theta \right)$$

$$= \lambda a (T_c - T_g) 4\pi$$

$$\phi_0 = 4\pi \lambda a (T_c - T_g)$$

$$q = j \cdot S$$

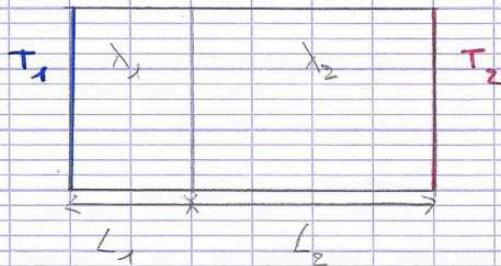
$$3) T_c - T_g = R_{th} \phi$$

$$R_{th} = \frac{1}{4\pi \lambda a}$$

$$s) R_{th} = \frac{1}{\pi \cdot 0,6 \cdot 10^{-3}} = 0,132 \cdot 10^8 \text{ W/K}$$

$$P_o = \frac{T_c - T_g}{R_{Th}} \approx 5,3 \mu W$$

Exercice 20



$$1) \vec{j} = -\lambda \vec{\nabla} T$$

$$\frac{dT}{dx} = K \Delta T$$

$\frac{dT}{dx} \rightarrow \text{stat donc } \Delta T = 0$

$$2) \vec{j}_1 = -\lambda_1 \frac{dT_1}{dx} \vec{e}_x$$

$$\frac{d^2 T(x)}{dx^2} = 0$$

$$\vec{j}_2 = -\lambda_2 \frac{dT_2}{dx} \vec{e}_x$$

$$\begin{cases} T_1(x) = a_1 x + b_1 \\ T_2(x) = a_2 x + b_2 \\ b_1 = b_2 = T_0 \end{cases}$$

$$\Rightarrow \begin{cases} T_1(x) = a_1 x + T_0 \\ T_2(x) = a_2 x + T_0 \end{cases}$$

$\Downarrow$

Conditions limites

$$T_1(-L_1) = T_1$$

$$T_2(+L_2) = T_2$$

$$T_1(0) = T_0$$

$$T_2(0) = T_0$$

$$j_1(0) = j_2(0)$$

$$\begin{cases} T_1(x) = \frac{T_0 - T_1}{L_1} x + T_0 \\ T_2(x) = \frac{T_2 - T_0}{L_2} x + T_0 \end{cases}$$

$$3) \vec{j}_1 = -\lambda_1 \frac{dT_1}{dz} \vec{e}_x$$

$$\vec{j}_2 = -\lambda_2 \frac{T_0 - T_1}{L_2} \vec{e}_x \quad \text{idem} \quad \vec{j}_2 = -\lambda_2 \frac{T_2 - T_0}{L_2} \vec{e}_x$$

$$\Rightarrow T_0 = \frac{\lambda_1/L_1 T_1 + \lambda_2/L_2 T_2}{\lambda_1/L_1 + \lambda_2/L_2}$$

$$4) \left. \begin{aligned} G_1 &= \frac{\lambda_1}{L_1 S} = \frac{1}{R_1} \\ G_2 &= \frac{\lambda_2}{L_2 S} = \frac{1}{R_2} \end{aligned} \right\} T_0 = \frac{G_1 T_1 + G_2 T_2}{G_1 + G_2}$$

$$5) L_1 = L_2 \\ T_0 = \frac{\lambda_1 T_1 + \lambda_2 T_2}{\lambda_1 + \lambda_2}$$

6) con bois

$$\lambda_1 = \lambda_{\text{peau}} = 10 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$$

$$\lambda_2 = \lambda_{\text{bois}} = 1 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$$

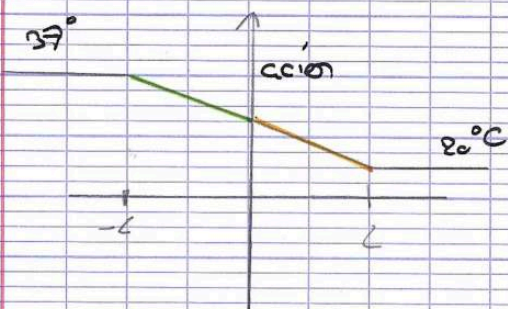
$$T_0^{\text{bois}} = \frac{10 \times 37 + 1 \times 20}{11} \approx 35,5^\circ \text{C}$$

con action

$$\lambda_1 = \lambda_{\text{peau}}$$

$$\lambda_2 = \lambda_{\text{action}} = 100 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$$

$$T_0 = \frac{10 \times 37 + 100 \times 20}{110} \approx 21,5^\circ \text{C}$$

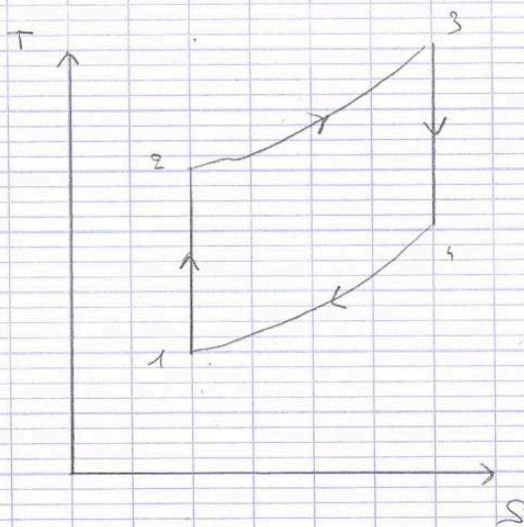
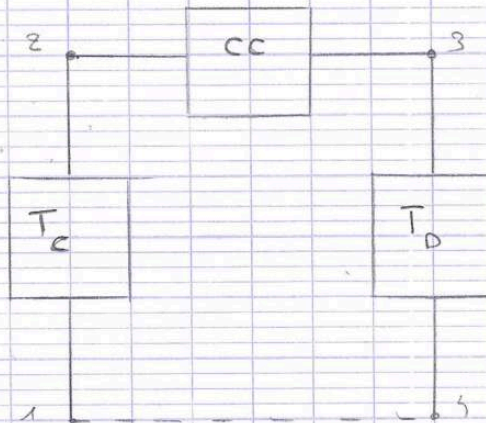


$$7) j = j_1 = j_2 \quad \text{em } X=0$$

$$j = \lambda_1 \frac{T_1 - T_0}{L}$$

$$j = \frac{\lambda_1}{L} \left(T_1 - \frac{T_1 - T_0}{L} \right)$$

Exercice 15



- 1 → 2 : isobaric
- 2 → 3 : isobaric réversible
- 3 → 4 : isobaric
- 4 → 1 : isobaric

1	2	3	4
P	1	6,5	6,5
T	300	T _c	1300
S	0	0	S ⁺

$$s_2 = s_1 = s_3 = 0 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$$

$$s_3 = s_1 + c_p \ln \left(\frac{T_3}{T_1} \right) - \frac{R}{M} \ln \left(\frac{P_3}{P_1} \right)$$

$$= s_2 + c_p \ln \left(\frac{T_3}{T_2} \right) - \frac{R}{M} \ln \left(\frac{P_3}{P_2} \right)$$

$$\rightarrow s_3 = s_2 + c_p \ln \left(\frac{T_3}{T_2} \right) - \frac{R}{M} \ln \left(\frac{P_3}{P_2} \right)$$

$$s_3 = c_p \ln \left(\frac{T_3}{T_2} \right) = 931,5 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$$

$$2) \Delta(h + e_p + e_c) = w + q$$

$$\Delta h + \Delta e_p + \Delta e_c = w + q$$

$$\text{ici } \Delta h = w + q$$

1 → 2

$$\Delta h_{12} = h_2 - h_1 = w_{12} + q_{12}$$

0 car adiabatique

$$\Rightarrow w_{12} = \Delta h_{12}$$

$$g.p. \rightarrow T_1 \rightarrow T_2$$

$$w_{12} = c_p (T_2 - T_1) = 212 \text{ RJ} \cdot \text{kg}^{-1}$$

$$w_{31} = c_p (T_1 - T_3) = -535 \text{ RJ} \cdot \text{kg}^{-1}$$

$$w_{\text{tot}} = w_{12} + w_{31} = -323 \text{ RJ} \cdot \text{kg}^{-1}$$

< 0 donc moteur.

$$3) \Delta h_{23} = w_{23} + q_{23}$$

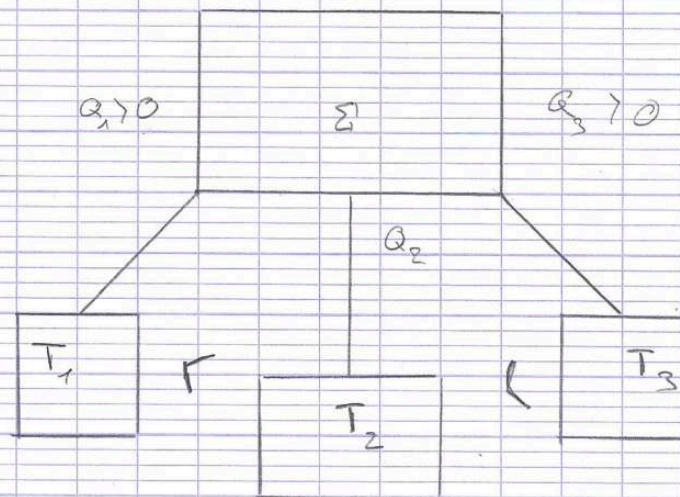
$$\rightarrow q_{23} = c_p (T_3 - T_2) = 788 \text{ RJ} \cdot \text{kg}^{-1}$$

$$4) e = \frac{w}{q} = \frac{-w_{\text{tot}}}{q_c} \approx 0,51$$

~~$$\Rightarrow w_{31} = c_p (T_1 - T_3) = -535 \text{ RJ}$$~~

~~$$w_{\text{tot}} = w_{12} + w_{31} = -323$$~~

Exercice 13



$Q_1 > 0$ car but du frigo

$$2) \text{ cycle } \Rightarrow \Delta U = 0 \Rightarrow w + Q_1 + Q_2 + Q_3 = 0$$

$$Q_1 + Q_2 + Q_3 = 0$$

$$\Sigma \frac{Q_i}{T_i} \leq 0 \Rightarrow \frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \frac{Q_3}{T_3} \leq 0$$

$$\Rightarrow -\frac{Q_3}{T_3} \geq \frac{Q_1}{T_1} + \frac{Q_2}{T_2} \quad \Leftrightarrow \quad Q_2 = Q_3 - Q_1$$

$$\Rightarrow -\frac{Q_3}{T_3} \geq \frac{Q_1}{T_1} - \frac{Q_3}{T_2} - \frac{Q_2}{T_2}$$

$$Q_3 \left(\frac{1}{T_2} - \frac{1}{T_3} \right) \geq Q_1 \left(\frac{1}{T_1} - \frac{1}{T_2} \right)$$

$$Q_3 \geq Q_1 \frac{\frac{1}{T_1} - \frac{1}{T_2}}{\frac{1}{T_2} - \frac{1}{T_3}}$$

$$3) \quad e = \frac{Q}{d} = \frac{Q_1}{Q_3}$$

$$e \leq e_{\max} = \frac{\frac{1}{T_2} - \frac{1}{T_3}}{\frac{1}{T_1} - \frac{1}{T_2}}$$

$$1) \quad e_c = \frac{T_8}{T_c - T_8} = \frac{\frac{1}{T_c}}{\frac{1}{T_8} - \frac{1}{T_c}} = 18,5$$

$$\text{alors } e_{\max} = \frac{\frac{1}{T_c}}{\frac{1}{T_8} - \frac{1}{T_c}} - \frac{\frac{1}{T_3}}{\frac{1}{T_8} - \frac{1}{T_c}} \leq e_c = 14,0$$

$$e_{\max} \leq e_c$$