Thermodynamique

Changements d'états

Exercice 1

eau à 100°C	h	u	s	v
V	2676	~~1,6729~~ 2506,5	7,35	1,6729
L	419	418,94		

1) • $h_{vap}(100^\circ) = h_v - h_L$

$= 2257 \text{ kJ/kg}$

• $s_{1\to 2}(T_0) = \dfrac{h_{1\to 2}(T_0)}{T_0}$ ici $s_{vap}(100^\circ C) = \dfrac{h_{vap}(100^\circ C)}{373,15}$

$= 6,05 \text{ kJ.kg}^{-1}.K^{-1}$

or $s_{vap} = s_v - s_L$

$s_L = 1,30 \text{ kJ kg}^{-1} K^{-1}$

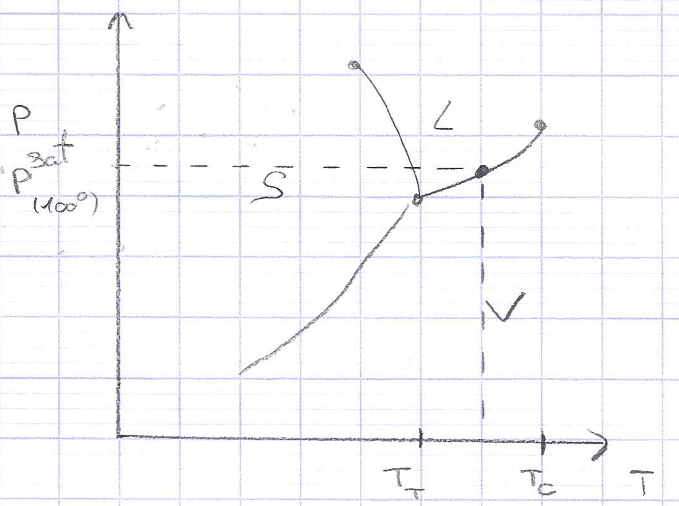
2) $u_{vap} = u_v - u_L = 2087,56 \text{ kJ.kg}^{-1}$

$h_{vap} = u_{vap} + P^{sat}(T)(v_v - v_L)$

$v_L = v_v - \dfrac{h_{vap} - u_{vap}}{P^{sat}(T)} = 0,0011 \text{ m}^3/\text{kg}$

or $\rho_L = \dfrac{1}{v_L} = 909 \text{ kg/m}^3 \Rightarrow$ Faux car les mesures montrent que $\rho_L(100^\circ, 1 \text{atm}) = 961 \text{ kg.m}^{-3}$

3)

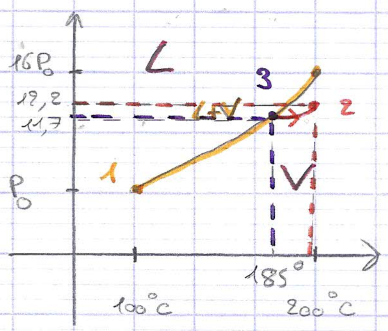


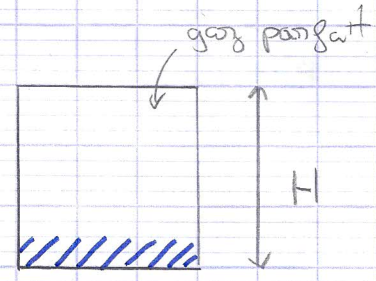
4) $\left(\frac{\partial P}{\partial T}\right)_{T_0} = \frac{h_{vap}(T_0)}{T_0(v_v - v_L)} = \frac{2257.10^3}{373(1{,}6729 - 0{,}0011)} \simeq 3620 \; Pa/K$

~~Météo~~ ~~$\pm 1°C$~~ ~~sur~~ $100°C$ eau celsius

5) $\left(\frac{\partial P}{\partial T}\right)_{0°C} = \frac{h_{fus}(0°C)}{T(v_L - v_S)} \simeq -1{,}35 \cdot 10^{7} \; Pa/K$
$\simeq -135 \; bar/K$

Exercice 3 : Eau liquide et eau vapeur dans une chaudière

1) 

2) 

$PV = m_{v_1} R(\theta_1 + 273)$

$V = (H-h)S$

$V \sim HS$ car $h \ll H$

équation liq_1 vap_1 $\theta = \theta_1 = 100°C$

$\Rightarrow P = P^s(\theta_1) = P_0$

$$\boxed{m_{v_1} = \frac{P_0 SH}{R(\theta_1 + 273)}}$$

$= 32,1$ mol

3) $m_{l_1} = ?$

$M = 18 \text{ g.mol}^{-1}$

$\rho = 1 \text{g.cm}^{-3}$

$m = \rho V = \rho S h$

$$\boxed{m_{l_1} = \frac{m}{M} = \frac{\rho S h}{M}}$$

$m_{l_1} = \cancel{277},8$ mol

4) $$m_{v_1} = \frac{P^s(\theta_1) SH}{R(\theta_1 + 273)}$$

$$= \frac{\left(\frac{\theta_1}{100}\right)^4}{(\theta_1 + 273)} \cdot \frac{P_0 SH}{R}$$

m_v devient donc

$$m_v \simeq \frac{P_0 SH}{R} \cdot \frac{(\theta/100)^4}{\theta + 273}$$

5) état 2, $\theta_2 = 200°C$, $m_{liq2} = 0$

$\Rightarrow m_{vap2} = m_{tot} = m_{vap1} + m_{liq}$

$P_2 V = m_{tot} R(\theta_2 + 273)$

$$\Rightarrow \boxed{P_2 = \frac{(m_{vap1} + m_{liq1}) R(\theta_2 + 273)}{SH}}$$

$= 12,2$ bars

6) Dernière goutte de liquide ③

$m_{liq3} = 0$ mais $P_3 = P^{sat}(\theta_3)$

$m_{vap3} = 0$

$\Rightarrow$ que du gaz $\Rightarrow P_3 V = m_{tot} R(\theta_3 + 273)$

$P_0 \left(\frac{\theta_3}{100}\right)^4 SH = (m_{liq} + m_{vap}) R(\theta_3 + 273)$

On a donc un polynôme de la forme $a\theta^4 + b\theta + c = 0$

3.

$$m_{tot} = m_v = \frac{P_0 S H}{R} \frac{(\theta/100)^4}{\theta + 273}$$

$\Rightarrow \theta_3 = 185\ °C$

$P^s(\theta_3) = 11,7$ bars

Exercice 4 : détente d'une vapeur saturante sèche

détente : $V \nearrow$

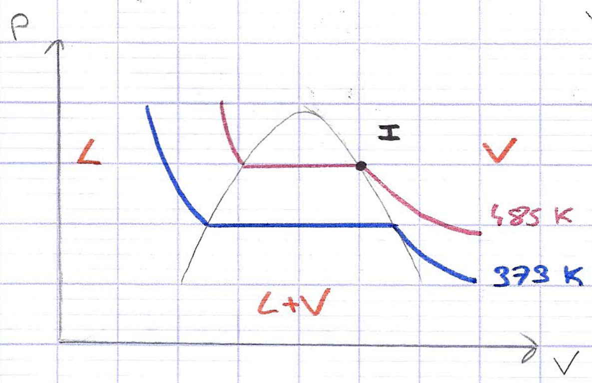
$T_1 \to T_2$

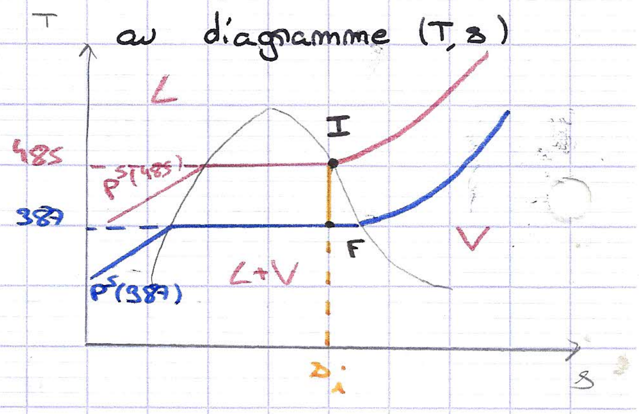
$485 K \to 373 K$

vapeur saturante sèche $\nearrow x = 1$

↳ limite du liquide

on passe donc au diagramme (T, s)





$v_i = \frac{V_i}{m}$

$v_f = \frac{V_f}{m}$

la transformation est une isentropie $\Rightarrow s = cte$

$x_i = 1$ on a 100 % de vapeur

$$x = \frac{LM}{LV} = \frac{s(M) - s(L)}{s(V) - s(L)} = 0,83$$ donc 83 % de vapeur

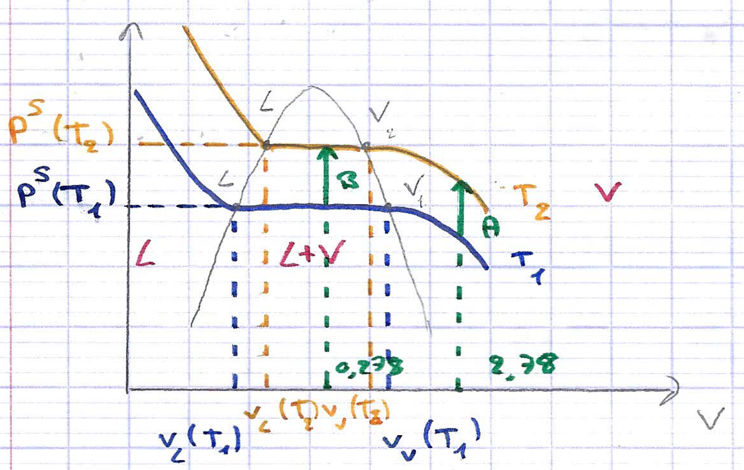
$$x = \frac{LM}{LV} = \frac{v(m) - v_L}{v_V - v_L}$$

$v(M) = x\, v_V + (1 - x) v_L$

$\Rightarrow v_f = 1,416\ m^3.kg^{-1}$

Exercice 6 : Chauffage d'une marmite

1)



2) $v_A = \frac{V_A}{m_A} = 2{,}78\ m^3.kg^{-1}$ $\quad x_A = 1$ $\quad$ vapeur

$v_B = \frac{V_B}{m_B} = 0{,}278\ m^3.kg^{-1}$ $\quad x_B = \frac{LB}{LV} = 16\ \%$

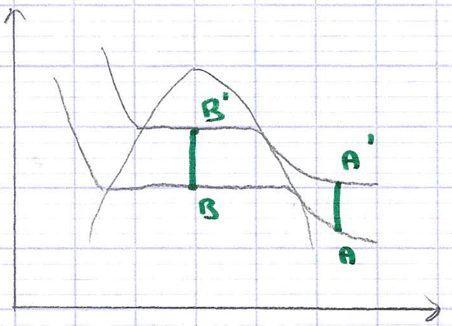
Transformation $T_A \rightarrow T_B$

3) a) $v = cte$ schéma $\Rightarrow$ A' vapeur

B' liq + vap

$$x_B' = \frac{v - v_L(T_2)}{v_V(T_2) - v_L(T_2)} = 0{,}71\ \%$$

b)



c) On cherche Q :

$Q = \Delta U$

$= \Delta U_A + \Delta U_B$

• $A \rightarrow A'$: vapeur → g.p → $\Delta U_{A'A} = C_v (T_2 - T_1)$

$= m_A C_v (T_2 - T_1)$

• $B \rightarrow B'$ $\quad \Delta U_{B'B} = \Delta U_{BV_1} + \underbrace{\Delta U_{V_1V_2}}_{= m_B c_v (T_2 - T_1)} + \Delta U_{V_2B'}$

$\Delta U_{BV_1} = m_B \Delta u_{BV_1}$

$= m_B \int_B^{V_1} du$

$dh = du + p^{sat} dv$

$\Rightarrow du = dh - p^{sat}(T)\, dv$

$\Rightarrow \Delta u_{BV_1} = \int_B^{V_1} dh - \int_B^{V_1} p^{sat}(T_1)\, dv$

or $dh = h_{1\rightarrow 2}(T)\, dx_2$

$= h_{vap}(T)\, dx$

$$\Delta u_{BV_1} = \int_{x_B}^{1} h_{vap}(T_1)\, dx - p^{sat}(T_1) \int_{v_B}^{v(v_1)} dv$$

$$\Delta u_{BV_1} = h_{vap}(T_1)\left[1 - x_B\right] - p^{sat}(T_1)\left[v_v(T_1) - v(B)\right]$$

$$\Delta u_{V^2B} = h_{vap}(T_2)\left[x_B' - 1\right] - p^{sat}(T_2)\left[v(B') - v_v(T_2)\right]$$

donc :

$$\Delta U = m_A c_v (T_2 - T_1) + m_B c_v (T_2 - T_1) + m_B h_{vap}(T_1)\left[1 - x_B\right] + m_B h_{vap}(T_1)\left[x_B' - 1\right] - m_B p^{sat}(T_1)\left[v_v(T_1) - v_B\right] - m_B p^{sat}(T_2)\left[v(B') - v_v(T_2)\right]$$

d) $\Delta S = \Delta S_{A \to A'} + \Delta S_{B \to B'}$

$du = T ds - P dV$

$ds = \frac{1}{T} du + \frac{P}{T} dV$

gaz parfait $\begin{cases} du = C_v dT \\ \frac{P}{T} = \frac{mR}{V} \end{cases}$

$\Rightarrow ds = Cv \frac{dT}{T} + mR \frac{dV}{V}$

$\Delta S = \int_A^{A'} ds = \int_{T_1}^{T_2} Cv \frac{dT}{T} + \int_{V_A}^{V_{A'}} m\ldots$

$\Delta S_{AA'} = C_v \ln\left(\frac{T_2}{T_1}\right)$

$\Delta S_{AA'} = m_A c_v \ln\left(\frac{T_2}{T_1}\right)$

- $\underline{B \to B'}$ en passant par L_1 et L_2

$\Delta S_{BB'} = \Delta S_{BL_1} + \Delta S_{L_1 L_2} + \Delta S_{L_2 B'}$

$\Delta S_{L_1 + L_2} =$ facile

$dh = c dT$

$ds = \frac{dh}{T} = c \frac{dT}{T}$

$\Delta S_{L_1 L_2} = m_B \cdot c \cdot \ln(T_2/T_1)$

$ds_{1 \to 2} = \frac{dh_{1 \to 2}}{T_0}$ donc $\Delta s_{BL_1} = \int_B^{L_1} \frac{h_{vap}(T_1)}{T_1} dx$

$\Rightarrow \Delta s_{BL_1} = \frac{h_{vap}(T_1)}{T_1} (-x_B)$

$\Delta s_{BL_2} = \frac{h_{vap}(T_2)}{T_2} (x_B')$

Bilan

$\Delta S = m_A \cdot c_v \ln(T_2/T_1) + m_B c_{liq} \ln(T_2/T_1)$

$+ m_B \left[\frac{h_{vap}(T_2) x_B'}{T_2} - \frac{h_{vap}(T_1) x_B}{T_1} \right]$

$= 5535,65 \text{ J.K}^{-1}$

2^{nd} principe :

$$dS_{source} = \frac{\delta Q_{source}}{T_{source}} = \frac{-\delta Q}{T_{source}}$$

ici $T_{source} = T_2$

$$\Rightarrow \Delta S_{source} = -\int \frac{\delta Q}{T_2} = -\frac{1}{T_2}\int dQ$$

$$\Delta S_{source} = -\frac{Q}{T_2} = -5212 \ J.K^{-1}$$

or 2^{nd} principe $\Delta S = \Delta S_{ech} + \Delta S_{créé}$

$$\Rightarrow \Delta S = \frac{Q}{T_2} + S_{créé}$$

Donc $S_{créé} = \Delta S - \frac{Q}{T_2} = 323 \ J.K^{-1}$

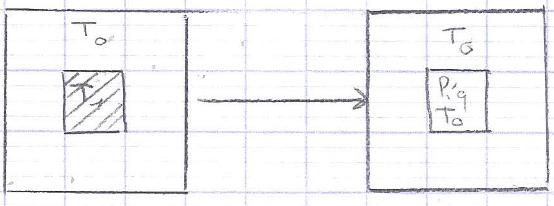
cas du système A :

$$S_{éch} = \frac{Q}{T_2} = \frac{mc_v(T_2 - T_1)}{T_2} = m_A c_v \left(1 - \frac{T_1}{T_2}\right)$$

$$\Delta S_A = m_A c_v \ln(T_2/T_1),$$

Exercice 7 : Transfert thermique et état métastable

1)



$\Delta H = \Delta H_1 + \Delta H_2$

$\Delta H_1 = ?$ corps solide $dh = c\,dT$

$\Rightarrow \Delta H = m\,\Delta h = m \int_i^f dh$

$= m \int c\,dT$

$\Delta H_1 = m_s\, c_{p,s}\,(T_0 - T_1)$

Changement d'état

$\Delta H_2 = m_s\, h_{fus}\,(T_0)$

$\Delta H = m_s\, c_{p,s}\,(T_1 - T_0) + m_s\, h_{fus}\,(T_0)$

$\Delta H = 375{,}8$ kJ

2) $\underset{m_s}{T_1} \xrightarrow{c_p^{sol}} \underset{m_s}{T_0} \xrightarrow{h_{fus}} \underset{m_s}{T_0} \xrightarrow{c_p^{liq}} \underset{m_s}{T_f}$

ici T_2 min tel que $T_f = T_0$ et glaçon fondu

$$\begin{array}{l} a \\ b \end{array}\left\{\begin{array}{l} \text{glace } T_1 \ m_s \xrightarrow{c_p^{sol}} \text{glace } T_0 \ m_s \xrightarrow{h_{fus}} \text{liq } m_s \ T_0 \\ \text{liq } T_2^{min} \ m_L \xrightarrow{c_p^{liq}} \text{liq } m_L \ T_0 \end{array}\right.$$

$\Delta H_a = m_s\, c_{p,s}\,(T_0 - T_1) + m_s\, h_{fus}$

$\Delta H_b = m_L\, c_p^{liq}\,(T_0 - T_2^{min})$

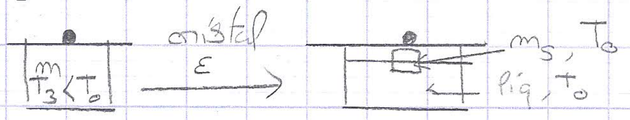
ici calorimètre isolé et capacité calorifique $= 0$

$\Delta H_{tot} = 0$

$$\Rightarrow \Delta H_a + \Delta H_b = 0 \Rightarrow T_2^{min} = T_0 + \frac{m_s C_p^{sol}(T_0 - T_1) + m_s h_{fus}}{m_L c_p^{liq}}$$

3) m

a) $T_3 < T_0$



$T_{finale} = T_0$ } équilibre liq + solide

b)

$$\begin{cases} a & \begin{array}{c} \text{liq} \\ m_s \\ T_3 \end{array} \xrightarrow{c_p^{liq}} \begin{array}{c} \text{liq} \\ m_s \\ T_0 \end{array} \xrightarrow{-h_{fus}} \begin{array}{c} \text{sol} \\ m_s \\ T_0 \end{array} \\ b & \begin{array}{c} \text{liq} \\ m - m_s \\ T_3 \end{array} \xrightarrow{c_p^{liq}} \begin{array}{c} \text{liq} \\ m - m_s \\ T_0 \end{array} \end{cases}$$

$\Delta H = \Delta H_a + \Delta H_b$

$\Delta H_a = C_p^{liq} m_s (T_0 - T_3) - m_s h_{fus}$

$\Delta H_b = (m - m_s) C_p^{liq} (T_0 - T_3)$

or $\Delta H = 0$

$\Rightarrow m\, c_p^{liq} (T_0 - T_3) - m_s h_{fus} = 0$

$$m_s = \frac{m\, c_p^{liq} (T_0 - T_3)}{h_{fus}}$$

c) $\Delta S^{réelle} = S_{éch}^{réelle} + S_{créé}^{réelle}$

j'immagine une transformation réversible ayant les mêmes états i et f.

$\Rightarrow \Delta S^{img} = S_{éch}^{img} + \cancel{S_{créé}^{img}} = \Delta S^{réelle}$

$\Rightarrow$ Je calcule $\Delta S^{img} = \Delta S^{img}_{ech}$.

$\Rightarrow \Delta S^{Reelle} = \Delta S^{img}_{e} + \Delta S^{img}_{b}$

$\Delta S^{img}_{b} = ?$ modèle liquide $ds = c\frac{dT}{T}$

$\Rightarrow \Delta S_b = (m - m_s)\int ds = (m - m_s)\int_{T_3}^{T_0} c\frac{dT}{T}$

$\Delta S_b = (m - m_s)\, c \ln\left(\frac{T_0}{T_3}\right)$

- $\Delta S^{img}_{a} = ?$

partie 1 pour liq $T_3 \to T_0$

$\Delta S^{1}_{a} = m_s\, c \ln\left(\frac{T_0}{T_3}\right)$

partie 2 "chgmt d'état"

$ds_{l\to s} = \frac{dh_{l\to s}}{T_0}$ $\Delta s_{l\to s} = \frac{\Delta h_{l\to s}}{T_0}$

$\Rightarrow \Delta S_a = \frac{\Delta H_{l\to s}}{T_0} = \frac{-m_s h_{fus}}{T_0}$

<u>Bilan</u>

$\Delta S^{img} = (m - m_s)\, c \ln\frac{T_0}{T_3} + m_s\, c \ln\frac{T_0}{T_s} - \frac{m_s h_{fus}}{T_0}$

$\Delta S^{img} = m\, c_p^{liq} \ln\left(\frac{T_0}{T_3}\right) - \frac{m_s h_{fus}}{T_0}$

<u>d)</u> $m_s = 0,125$ kg

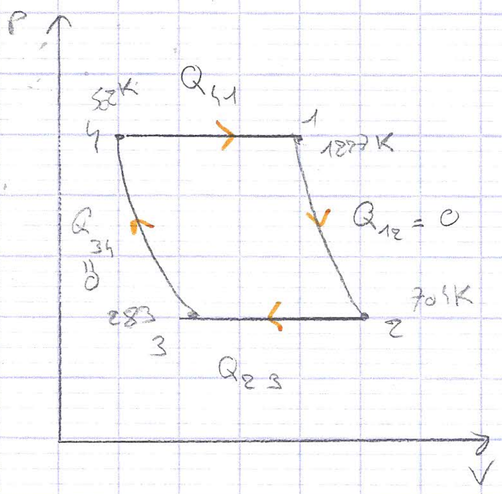
$\Delta S^{reel} = \Delta S^{img} = 3,05\ J.K^{-1}$

$= \underset{\substack{\| \\ 0 \text{ car système isolé}}}{S^{réel}_{ech}} + S^{réel}_{crée} \Rightarrow S^{réel}_{crée} = 3,05\ J.K^{-1}$ donc irréversible

Exercice 9

Isentropique $P \propto \frac{1}{V^\gamma}$

1)



	T	P
1	1227	7
2	?	1
3	288	1
4	?	7

2) $e = -\frac{W}{Q_c}$

$\Delta U = W + Q = 0$ $\qquad e = 1 + \frac{Q_f}{Q_c}$

$\Rightarrow W + Q_c + Q_f = 0$

3) $1 \to 2$ adiatique réversible = isentropique

$$\begin{cases} dS = \delta S_{ech} + \delta S_{créé} \\ dS = \frac{\delta Q}{T_S} + \delta S_{créé} \end{cases}$$

adiabatique $\Rightarrow \delta Q = 0$

$\Rightarrow Q_{12} = 0$

4) $\left.\begin{matrix} 2 \to 3 \\ 4 \to 1 \end{matrix}\right\}$ réversible + isobare.

$dH = TdS + VdP$ $\qquad$ isobare $dP = 0$ $\qquad dH = TdS$

2ème principe : $dS = \delta S_{ech} + \delta S_{créé}$

$= \frac{\delta Q}{T} + \delta S_{créé}$

$\Rightarrow TdS = \delta Q$

5) $dH = \delta Q$ sur $2 \to 3$ $\Rightarrow \int_2^3 \delta Q = \int_2^3 dH$

$4 \to 1$

$Q_{23} = \Delta H_{23}$

gaz parfaits $\Rightarrow \Delta H_{23} = C_p (T_3 - T_2)$

$Q_{23} = C_p (T_3 - T_2)$

idem $Q_{41} = C_p (T_1 - T_4) > 0$

6) $1 \to 2$ Loi de Laplace

$3 \to 4$

$T^{\gamma} P^{1-\gamma} = cte$

$T_1^{\gamma} P_1^{1-\gamma} = T_2^{\gamma} P_2^{1-\gamma}$

$\Rightarrow T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$

$T_2 = 1227 \left(\frac{1}{7}\right)^{\frac{0,4}{1,4}} = 704 \, K$

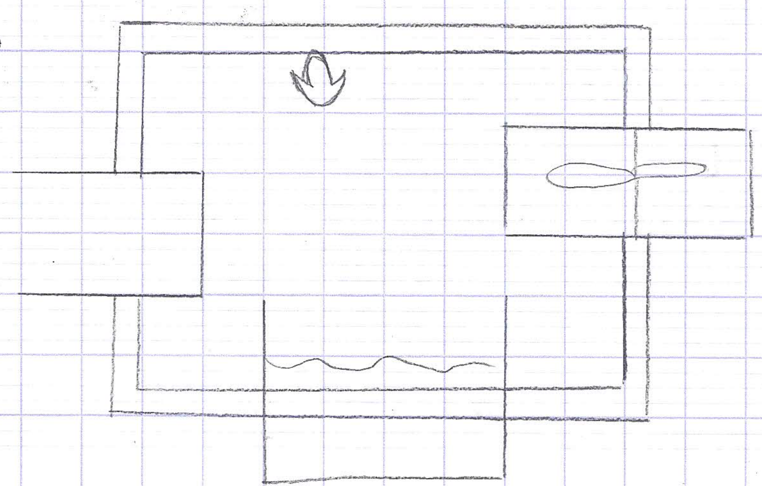
idem $T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} \simeq 502 \, K$

7) $Q_c > 0$ donc $Q_c = Q_{41}$ $(T_1 > T_4)$

$Q_f < 0$ donc $Q_f = Q_{23}$ $(T_3 < T_2)$

on $e = 1 + \frac{Q_f}{Q_c} = 1 + \frac{C_p (T_3 - T_2)}{C_p (T_1 - T_4)} \Rightarrow e = 1 - \frac{T_2 - T_3}{T_1 - T_4}$

8)

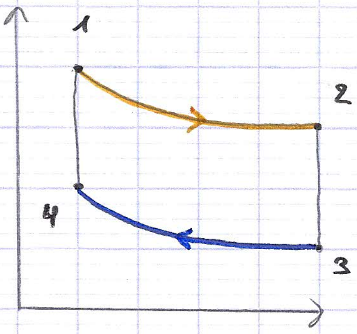


Exercice 10

1) Inconnues

P_1 P_2 $\quad PV = nRT$

$\Rightarrow n = 3{,}76 \cdot 10^{-2}$ mol



$W_{12} = -1{,}08\ kJ = -nRT_c \ln (V^+/V^-)$

$Q_{12} = 1{,}08\ kJ = -W_{12}$

$Q_c > 0$

$W_{23} = 0$

$Q_{23} = \Delta U_{23} = \frac{nR}{\gamma-1} (T_f - T_c) = -0{,}922\ kJ$

$W_{34} = nRT_f \ln(V^+/V^-) = 0{,}23\ kJ$

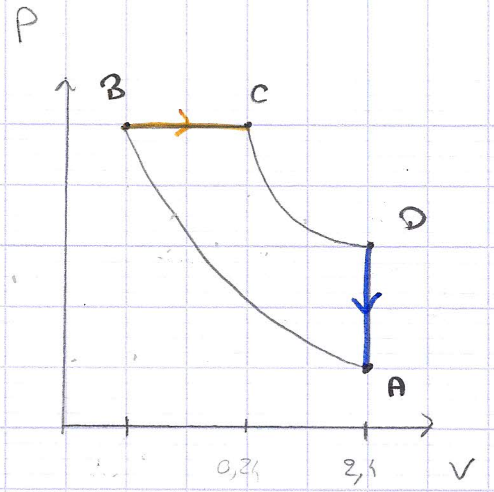
$Q_{34} = -W_{34} = -0{,}23\ kJ$

$e = \frac{-W}{Q_c} = 1 + \frac{Q_f}{Q_c} = 0{,}79 = e_c$ à cause de régénération.

Exercice 11

1)		A	B	C	D
bar	P	1	44,3	44,3	1,76
L	V	2,4	0,16	0,24	2,4
K	T	323	954	1431	570

3)



moteur ⟳

$TV^{\gamma-1} = c^{te}$

$V_B = V_A \left(\frac{T_A}{T_B}\right)^{\frac{1}{\gamma-1}}$

$= 0,16\ L$

$PV = mRT \Rightarrow P_B = 44,3$ bar

$PV = mRT \Rightarrow T_C = 1131\ K$

$TV^{\gamma-1} = c^{te}$ sur CD

$\Rightarrow T_D = 570\ K$

$P_D = 1,76$ bar

2) $C_v = \frac{mR}{\gamma-1} = 1,86\ J.K^{-1}$

$C_p = \gamma C_v = 2,6\ J.K^{-1}$

4) $A \rightarrow B$ syst fermée

$\Delta U_{AB} = W_{AB} + Q_{AB}$

→ adiabatique réversible $Q_{AB} = 0$

donc $W_{AB} = \Delta U_{AB}$

or $\Delta U_{AB} = C_v(T_B - T_A)$ car gaz parfait

$W_{AB} = C_v(T_B - T_A) = 1170\ J$

5) $\Delta U_{BC} = C_v(T_C - T_B) = W_{BC} + Q_{BC}$

$\Delta H_{BC} = C_p(T_C - T_B) = Q_{BC}$

$dU = TdS - PdV = \delta W + \delta Q$

$dH = TdS + VdP$

$dH = \delta W + \delta Q + pdV + VdP$

$Q_{BC} = C_p (T_C - T_B)$

$W_{BC} = C_V (T_C - T_B) - C_p (T_C - T_B)$

6) $W_{CD} = ?$ idem

$\begin{cases} Q_{CD} = ? \\ Q_{CD} = 0 \\ W_{CD} = C_V (T_D - T_C) \end{cases}$

7) isochore $\begin{cases} W_{DA} = 0 \\ \Delta U_{DA} = Q_{DA} \end{cases}$ donc $Q_{DA} = C_V (T_A - T_D)$

8) $W_{ABCD} = C_V (T_B - T_A) + C_V (T_C - T_B) - C_p (T_C - T_B) + C_V (T_D - T_C) + 0$

$= C_V (T_D - T_A) - C_p (T_C - T_B)$

$= -780 \text{ J}.$

9) $e = \frac{u}{d} = \frac{-W}{Q_C} = \frac{-W_{ABCD}}{Q_{BC}}$

$= 1 - \frac{C_V}{C_p} \left(\frac{T_D - T_A}{T_C - T_B} \right)$

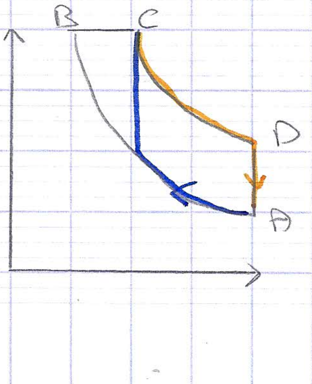
$= 1 - \frac{1}{\gamma} \frac{T_D - T_A}{T_C - T_B} = 0,63.$

10) $e_{max} = e_{carnot} = 1 - \frac{T_f}{T_c}$

$= 1 - \frac{T_A}{T_C} = 0,77$

irréversible

11)



12) $W_{AB_1} =$

$Q_{AB_1} =$

il faut $T_{B_1\gamma}$

$P_A V_A^\gamma = P_{B_1} V_{B_1}^\gamma$

$\Rightarrow P_{B_1} = 25,11$ bars

$T_{B_1} = 811$ K

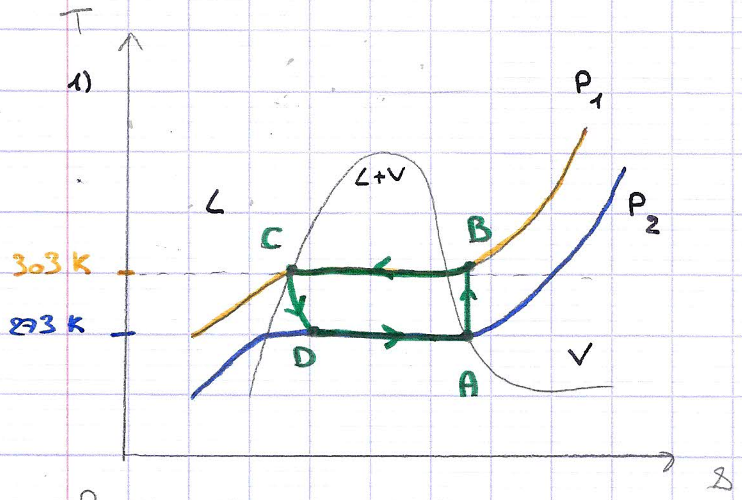
$\Rightarrow \begin{cases} W_{AB_1} = C_V (T_{B_1} - T_A) \\ Q_{AB_1} = 0 \end{cases}$

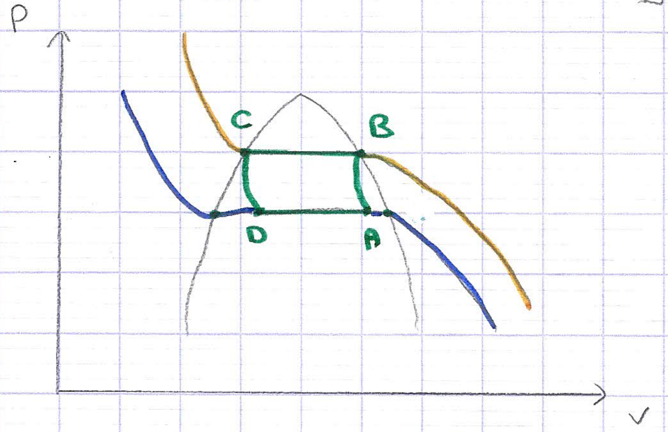
$\Rightarrow \begin{cases} W_{B_1C} = 0 \\ Q_{B_1C} = C_V (T_C - T_{B_1}) \end{cases}$

13) $e_{BdR} = 1 - \frac{T_D - T_A}{T_C - T_{B_1}} = 0,6.$

Exercice 12

1)





2) $\begin{cases} h_C = h_L(303\,K) \\ h_D = x_D h_V(273\,K) + (1 - x_D) h_L(273\,K) \end{cases}$

Transfo imaginaire

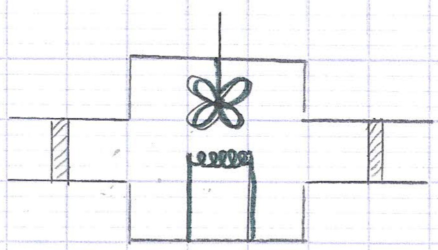
$\Delta h_{CD} = \Delta h_{CC'} + \Delta h_{C'D} = 0$

$= c(T_D - T_C) + h_{vap}(273\,K)(x_D - 0) = 0$

$x_D = \dfrac{c(T_C - T_D)}{h_{vap}(273K)} = 9{,}7\,\%$

3) 1er principe adaptés au système ouverts

$\Delta h = w_{autres} + q$



Compresseur calorifugé $\Rightarrow q = 0$

donc $w_{cp} = w_{autres} = \Delta h_{AB} = h_B - h_A$

gaz parfait $\Rightarrow \Delta h_{AB} = c_p (T_B - T_A)$

$\Rightarrow w_{cp} = c_p (T_B - T_A)$

$\Rightarrow W = m w_{cp} = m c_p (T_B - T_A)$

5) $e = \frac{u}{d} = \frac{q_f}{w} = \frac{q_{DA}}{w_{cp}}$ $\qquad e = \frac{h_{vap}(273\,K)(1 - x_D)}{c_p (T_B - T_A)}$

$q_{DA} = h_{vap}(273) [1 - x_D]$

$w_{cp} = c_p (T_B - T_A)$

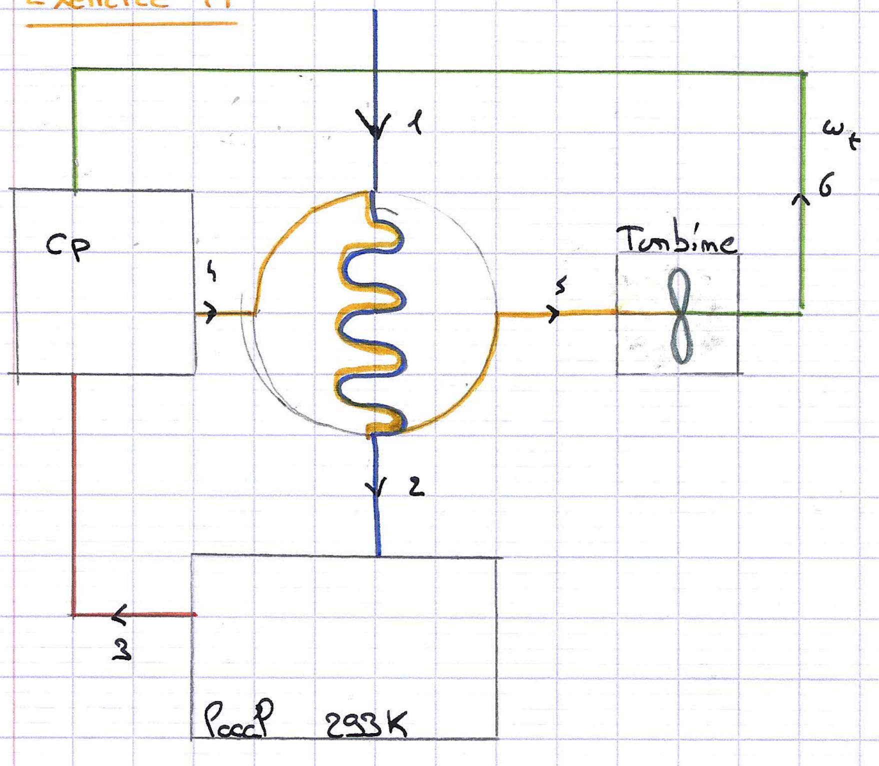
$A \to B$ isentropique $\quad PV^\gamma = cte \to P^{1-\gamma} T^\gamma = cte$

$T_B = T_A \left(\frac{P_A}{P_B}\right)^{\frac{1-\gamma}{\gamma}} = T_A \left(\frac{P_B}{P_A}\right)^{\frac{\gamma - 1}{\gamma}}$

$e = \frac{h_{vap}(273\,K)(1 - x_B)}{c_p T_A \left[\left(\frac{P_B}{P_A}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} = 7,5$

$e_c = \frac{T_F}{T_C - T_F} = 18,5.$

Exercice 11



1) 1 bar
-5°C

2) $T_2 = 351$ K
$P_2 = 1$ bar

3)

4) 2 bar
435 K

5) 2 bar
351 K

6) 1 bar
236 K

1) $\left.\begin{matrix} 3 \to 4 \\ 5 \to 6 \end{matrix}\right\}$ isoS $\to T^{\gamma} P^{1-\gamma} = cte$

2) $w_{34} = w_{cp} = \dfrac{\gamma R}{(\gamma-1)M} (T_4 - T_3)$

3) $w_{56} = \dfrac{\gamma R}{(\gamma-1)M} (T_6 - T_5)$

	1	2	3	4	5	6
P	1	1	1	4	4	1
T	268	351	293	435	351	236

2) $w_{34} = \frac{\gamma R}{(\gamma-1)M} (T_4 - T_1) = 142,5 \text{ kJ.kg}^{-1}$

3) $w_{56} = \frac{\gamma R}{(\gamma-1)M} (T_6 - T_5) = -115,4 \text{ kJ.kg}^{-1}$

4) $w_{tot} = w_{56} - w_{34} = 27,1 \text{ kJ.kg}^{-1}$

5) $q_c = q_{33} = \Delta h = \frac{\gamma R}{(\gamma-1)M} (T_3 - T_2) = -58,2 \text{ kJ.kg}^{-1}$

6) $e_{PAC} = \frac{u}{d} = \frac{-q_c}{w_{56} + w_{34}} = 2,2$

7) $e_c = \frac{T_c}{T_c - T_F} = 11,7$

Physique de la diffusion

Exercice 16 : Mesure d'une conductivité thermique.

1) $\vec{j}_{th} = -\lambda \vec{\nabla} T$ et z $\vec{j}_{th}(z) = -\lambda \frac{dT}{dz} \vec{e_z}$

$j_{th} = W.m^{-2}$

$\lambda = W.m^{-1}.K^{-1}$

2) $\varphi_{th} = \iint \vec{j}_{th}(\varphi) \vec{dS} = \iint j_{th}\, dS = j_{th}(z) \pi r^2$

3) $P = UI = RI^2$ donc R grand

4) $P = \varphi_{th}$

$\frac{E^2}{R} = j_{th}(z) \pi r^2$

$j_{th}(z) = \frac{E^2}{R \pi r^2}$

5) Bilan des flux pendant dt.

$dU_{in} = j(z,t)\, dS dt$

$dU_{out} = j(z+dz,t)\, dS dt$

$dU = (j(z,t) - j(z+dz,t))\, dS dt$

$U(t) = u(z,t)\, dm = u(z,t)\, dS dz \rho$

$U(t+dt) = u(z, t+dt)\, dS dz \rho.$

$dU = (\rho u(z,t+dt) - \rho u(z,t))\, dS dz$

$\frac{\partial \rho u}{dt} = -\frac{dj}{dz}$ stationnaire donc $\frac{\partial j}{\partial z} = 0$

$j_{th}(z) = cte$

$j_{th} = -\lambda \frac{dT}{dz} = cte$

$T(x) = az + b$

6) $\vec{j}_{th}(z) = j_{th}\,\vec{e}_z$

$$j_{th} = -\lambda \frac{dT}{dz} = -\lambda \frac{T_L - T_0}{L}$$

$$\vec{j}_{th} = \lambda \frac{T_0 - T_L}{L} \vec{e}_z$$

7) $T(z) = \dfrac{-j_{th}}{\lambda} z + T_0$

or $j_{th} = \dfrac{\mathcal{E}^2}{R\pi r^2}$

donc $T(z) = -\dfrac{\mathcal{E}^2}{R\pi r^2 \lambda} z + T_0$

$$= -\frac{T_L - T_0}{L} z + T_0$$

$$T(L) = -\frac{\mathcal{E}^2}{R\pi r^2 \lambda} L + T_0$$

$$T_0 = T_L + \frac{\mathcal{E}^2}{R\pi r^2 \lambda} L$$

8) $U = RI$

$\cancel{T_{th}} = R_{Th}\,\varphi$

$T_c - T_F$

$$T(z) = \frac{T_L - T_0}{L} z + T_0$$

$$j = \lambda \frac{T_0 - T_L}{L}$$

$$\varphi = j \cdot S = \frac{\lambda S}{L}(T_0 - T_L)$$

$$T_0 - T_L = \varphi \frac{L}{\lambda S}$$

$$R_{Th} = \frac{T_0 - T_L}{\varphi} = \frac{L}{\pi r^2 \lambda} = 11{,}1\ K.W^{-1}$$

$$r_{th} = \frac{R_{TH}}{L} = \frac{1}{\pi r^2 \lambda} = 27{,}8\ K.W^{-1}.m^{-1}$$

5) $T(z_1) = T_1$

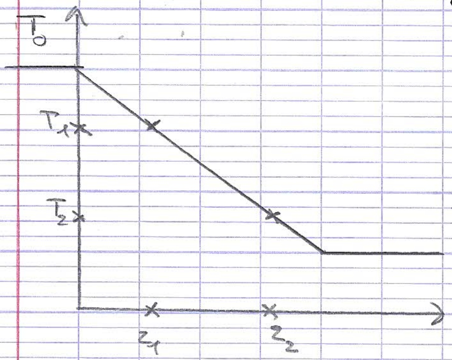
$T(z_2) = T_2$

$\text{pente} = \dfrac{T_1 - T_2}{z_1 - z_2}$ $\qquad \dfrac{1}{\lambda} = \dfrac{Rm^2}{\varepsilon^{-2}} \dfrac{T_1 - T_2}{z_1 - z_2}$

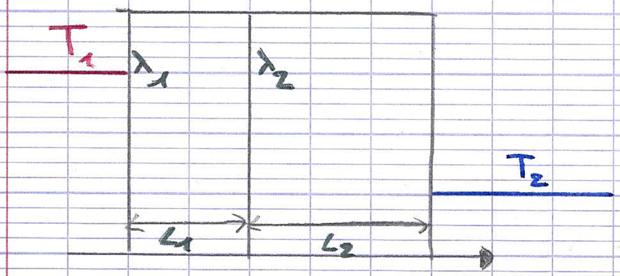
$j = -\lambda \times \text{pente}$

or $j = \dfrac{\varepsilon^2}{Rm^2}$

$\text{pente} = \dfrac{\varepsilon^2}{Rm^2}$



Exercice 17 : Diffusion de chaleur à travers une paroi composite



1) Régime stationnaire
↳ $\Delta T(x, y, z) = 0$
Problème 1D → $T(x)$

$$\frac{d^2 T(x)}{dx^2} = 0$$

donc $\begin{cases} T_1(x) = a_1 x + b_1 \\ T_2(x) = a_2 x + b_2 \end{cases}$ 4 inconnues.

2) Continuité de T
Continuité de j car stationnaire

4 conditions aux limites

$T_1(0) = T_1$ 1

$T_2(L_1 + L_2) = T_2$ 2

$T_1(L_1) = T_2(L_1)$ 3

$\vec{j_1}(L_1) = \vec{j_2}(L_2)$

on par Fourier $\vec{j}_1 = -\lambda_1 \dfrac{dT_1}{dx} \vec{e}_x$

$\vec{j}_2 = -\lambda_2 \dfrac{dT_2}{dx} \vec{e}_x$

$\Rightarrow \lambda_1 \dfrac{dT_1}{dx}(L_1) = \lambda_2 \dfrac{dT_2}{dx}(L_2)$ ④

① $\Rightarrow T_1(0) = b_1 = T_1 \Rightarrow b_1 = T_1$

② $\Rightarrow T_2(L_1+L_2) = a_2(L_1+L_2) + b_2 = T_2$

$a_2(L_1+L_2) + b_2 = T_2$

$\Rightarrow \boxed{T_1(x) = \dfrac{dT_1(x)}{dx} x + T_1} = a_1 x + T_1$

$T_2(x) = \dfrac{dT_2}{dx} \cdot x + (T_2 - a_2(L_1+L_2))$

$\boxed{T_2(x) = \dfrac{dT_2(x)}{dx}(x - (L_1+L_2)) + T_2} = a_2(x-(L_1+L_2)) + T_2$

④ $\Rightarrow \lambda_1 a_1 = \lambda_2 a_2 = \alpha$

donc $a_1 = \dfrac{\alpha}{\lambda_1}$ $a_2 = \dfrac{\alpha}{\lambda_2}$

$$\boxed{\begin{array}{l} T_1(x) = \dfrac{\alpha}{\lambda_1} x + T_1 \\ T_2(x) = \dfrac{\alpha}{\lambda_2}[x - (L_1+L_2)] + T_2 \end{array}}$$

③ $\Rightarrow T_1(L_1) = T_2(L_1)$

$\Rightarrow \dfrac{\alpha L_1}{\lambda_1} + T_1 = -\dfrac{\alpha L_2}{\lambda_2} + T_2$

$\Rightarrow \alpha\left(\dfrac{L_1}{\lambda_1} + \dfrac{L_2}{\lambda_2}\right) = T_2 - T_1$

$$\Rightarrow \alpha = \frac{T_2 - T_1}{\frac{L_1}{\lambda_1} + \frac{L_2}{\lambda_2}}$$

donc
$$\boxed{T_1(x) = \frac{1}{\lambda_1}\left(\frac{T_2 - T_1}{\frac{L_1}{\lambda_1} + \frac{L_2}{\lambda_2}}\right) x + T_1 \qquad T_2(x) = \frac{1}{\lambda_2}\left(\frac{T_2 - T_1}{\frac{L_1}{\lambda_1} + \frac{L_2}{\lambda_2}}\right)(x - (L_1 + L_2)) + T_2}$$

3) $$T_i = T_1(L_1) = \frac{1}{\lambda_1}\,\frac{T_2 - T_1}{\frac{L_1}{\lambda_1} + \frac{L_2}{\lambda_2}}\, L_1 + T_1$$

$$R = \frac{L}{\lambda S} \qquad G = \frac{\lambda S}{L} \qquad \begin{cases} G_1 = \frac{\lambda_1}{L_1} S \\ G_2 = \frac{\lambda_2}{L_2} S \end{cases}$$

$$T_i = \frac{L_1}{\lambda_1}\left(\frac{T_2 - T_1}{\frac{L_1 S_1}{\lambda_1 S_2} + \frac{L_2 S_2}{\lambda_2 S_2}}\right) = \frac{L_1}{\lambda_1 S_1}\,\frac{T_2 - T_1}{\frac{L_1}{\lambda_1 S_2} + \frac{L_2}{\lambda S_2}}$$

$$= \frac{1}{G_1}\,\frac{T_2 - T_1}{\frac{1}{G_1} + \frac{1}{G_2}} + T_1$$

$$= \frac{(T_2 - T_1) G_2}{G_1 + G_2} + \frac{T_1 (G_1 + G_2)}{G_1 + G_2}$$

$$\boxed{T_i = \frac{G_1 T_1 + G_2 T_2}{G_1 + G_2}}$$

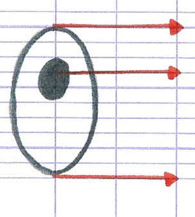
4) $$\vec{j}_1 = -\lambda_1 \frac{dT_1}{dx}\vec{e}_x$$

$$j = j_1 = j_2 = \frac{T_1 - T_2}{\frac{L_1}{\lambda_1} + \frac{L_2}{\lambda_2}}$$

$$j_1 = -\lambda_1\left(\frac{1}{\lambda_1}\,\frac{T_2 - T_1}{\frac{L_1}{\lambda_1} + \frac{L_2}{\lambda_2}}\right)$$

$$\varphi = \iint \vec{j} \cdot \vec{dS} = \iint j\vec{e}_x \, dS\vec{e}_x$$
$$= j \iint dS$$
$$= jS$$

$$j_2 = -\lambda_2\left(\frac{1}{\lambda_2}\,\frac{T_2 - T_1}{\frac{L_1}{\lambda_1} + \frac{L_2}{\lambda_2}}\right)$$

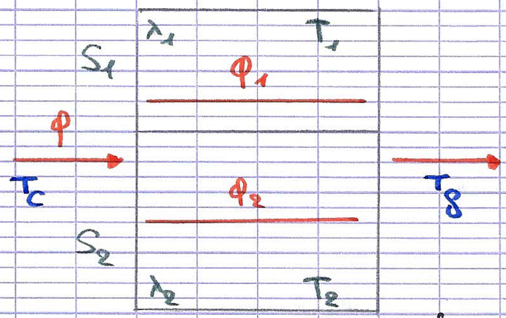


$U = R_{eq} I$

$-T_2 + T_1 = \left(\frac{L_1}{\lambda_1 S} + \frac{L_2}{\lambda_2 S} \right) \varphi$

$R_{eq} = \frac{L_1}{\lambda_1 S} + \frac{L_2}{\lambda_2 S} = R_1 + R_2$

5)



$T_1(x) = a_1 x + b_1$

$T_2(x) = a_2 x + b_2$

$$\begin{cases} T_1(x) = \frac{T_8 - T_c}{L} x + T_c \\ T_2(x) = \frac{T_8 - T_c}{L} x + T_c \end{cases}$$

$\vec{j}_1 = -\lambda_1 \frac{dT_1}{dx} \vec{e}_x = \lambda_1 \left(\frac{T_c - T_8}{L} \right) \vec{e}_x$

$\vec{j}_2 = -\lambda_2 \frac{dT_2}{dx} \vec{e}_x = \lambda_2 \left(\frac{T_c - T_8}{L} \right) \vec{e}_x$

$\varphi_1 = \iint \vec{j}\, \vec{dS} = \frac{\lambda_1 S}{L} (T_c - T_8)$

$\varphi_2 = \frac{\lambda_2 S}{L} (T_c - T_8)$

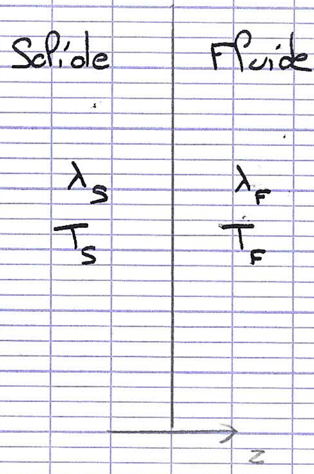
$\varphi_{tot} = \varphi_1 + \varphi_2$ $\qquad \varphi_{tot} = \left(\frac{\lambda_1 S}{L} + \frac{\lambda_2 S}{L} \right) (T_c - T_8)$.

$U = RI \Rightarrow I = \frac{U}{R}$ $\qquad \Rightarrow \frac{1}{R_{eq}} = \frac{\lambda_1 S}{L} + \frac{\lambda_2 S}{L}$

$\varphi_{tot} = \frac{1}{R_{eq}} (T_c - T_8)$ $\qquad \Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

Transfert conducto-convectif

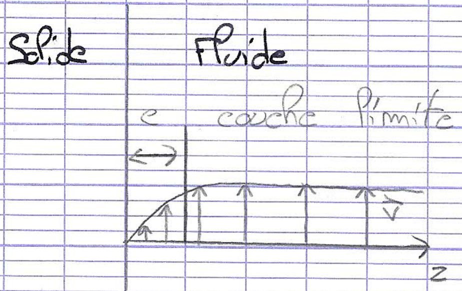
Cas lorsque l'une des interfaces fait intervenir un fluide (liquide ou gaz)

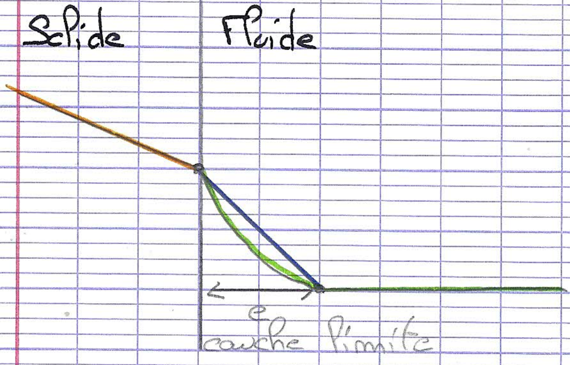


Continuité de $\vec{j}$ à l'interface

$\vec{j_S} = -\lambda_S \frac{dT_S}{dz}\vec{e_z}$ $\quad \vec{j_F} = -\lambda_F \frac{dT_F}{dz}\vec{e_z}$

$$\lambda_S \frac{dT_S}{dz} = \lambda_F \frac{dT_F}{dz}$$



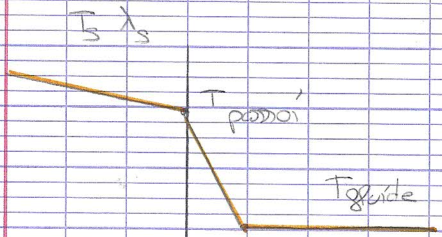


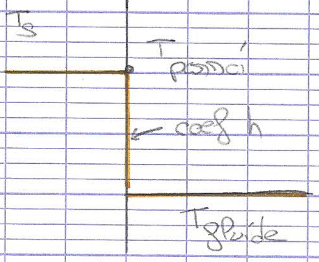
$$\vec{j_F} = -\lambda_F \frac{T_F - T_S}{e}\vec{e_z}$$

$$\vec{j} = -\frac{\lambda_F}{e}(T_{fluide} - T_{paroi})\vec{e_z}$$

$$= h(T_{paroi} - T_{fluide})\vec{e_z}$$

$h = \frac{\lambda_F}{e}$ coef de transfert conducto-convectif.

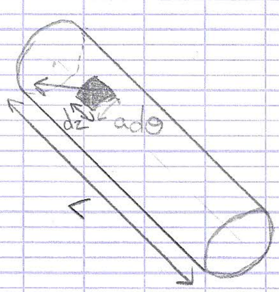




Exercice 18 : Isolation d'une canalisation d'eau chaude.

18.1

1)



$$\varphi = \iint \vec{j}\, \vec{dS}$$

$$\vec{dS} = a\,d\theta\, dz\, \vec{e_r}$$

$$\vec{j} = h(T_1 - T_0)\vec{e_r}$$

$$\varphi = \iint h_i (T_1 - T_0)\, a\, d\theta\, dz$$

$$\varphi = h(T_1 - T_0)\, a \int_0^L dz \int_0^{2\pi} d\theta$$

$$\varphi_0 = h(T_1 - T_0)\, 2\pi L a$$

2) $$(T_1 - T_0) = \frac{1}{2\pi L a h}\varphi_0$$

$$R_{Th} = \frac{1}{2\pi a L h} = \frac{1}{Sh}$$

18.2

1) $\Delta T = 0$

$T(r, \theta, z)$

$\rightarrow T(r)$

2) $$\Delta g(r) = \frac{1}{r}\frac{d}{dr}\left(r\frac{dg}{dr}\right)$$

$$\Delta T = 0 \Leftrightarrow \frac{1}{r}\frac{d}{dr}\left(r\frac{dg}{dr}\right) = 0$$

$$\Rightarrow r\frac{dT}{dr} = A$$

$$\frac{dT(r)}{dr} = \frac{A}{r} \Rightarrow T(r) = A \ln r + B$$

2 conditions limites $\begin{cases} T(r=a) = T_1 \\ T(r=L) = T_0 \end{cases}$

$$\begin{cases} T_1 = A \ln a + B \\ T_2 = A \ln b + B \end{cases}$$

$$A \ln b - A \ln a = T_2 - T_1$$

$$A = \frac{T_2 - T_1}{\ln(b/a)}$$

$$T(r) = \frac{T_2 - T_1}{\ln(b/a)} \ln(r/a) + B$$

$$T(r) = \frac{T_2 - T_1}{\ln(b/a)} \ln(r/a) + T_1$$

3) $\vec{j} = -\lambda \vec{\nabla} T = -\lambda \begin{vmatrix} \partial T/\partial r \\ 1/r\, \partial T/\partial \theta \\ \partial T/\partial z \end{vmatrix}$

$$T(r) \Rightarrow \vec{j} = -\lambda \frac{dT(r)}{dr} \vec{e_r}$$

$$\frac{dT}{dr} = \frac{T_2 - T_1}{\ln(b/a)} \cdot \frac{1}{r}$$

$$\vec{j}(r) = -\lambda \frac{T_2 - T_1}{\ln(b/a)} \frac{1}{r} \vec{e_r}$$

$$\vec{j}(a) = -\lambda \frac{T_2 - T_1}{\ln(b/a)} \frac{1}{a} \vec{e_r}$$

4) $\varphi = \iint \vec{j}\, \vec{dS}$

$\varphi(a) = \iint \vec{j}(a) . \vec{dS}$

$= \iint \vec{j}(a) . a\, d\theta\, dz\, \vec{e_r}$

$$\varphi(a) = \int_0^{2\pi} \int_0^{L} \lambda \frac{T_1 - T_2}{\ln(b/a)} \frac{1}{a} a\, d\theta\, dz$$

$$\varphi_1 = \varphi(a) = \lambda \frac{T_1 - T_2}{\ln(b/a)} 2\pi L$$

5) $\varphi_1 = \lambda \dfrac{T_1 - T_2}{\ln(b/a)} 2\pi L$

$\varphi_2 = 2\pi L\, b . h\,(T_2 - T_0)$

6) pas d'accumulation de chaleur donc $\varphi_1 = \varphi_2$

φ_1 = flux en $r = a$ ou $r = b$

7) $T_1 - T_2 = \dfrac{\lambda\,(T_1 - T_2)}{\cancel{bh \ln(b/a)}}\quad T_1 - T_0 + T_0 - T_2$

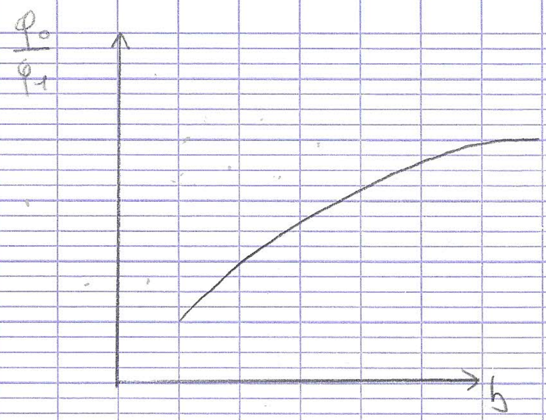
$$T_1 - T_2 = \frac{T_1 - T_0}{1 + \dfrac{\lambda}{bh \ln(b/a)}}$$

8) $\dfrac{\varphi_0}{\varphi_1} = \dfrac{a}{b} + \dfrac{ah}{\lambda} \ln\left(\dfrac{b}{a}\right)$

9) si $h = 0$ $\quad \dfrac{\varphi_0}{\varphi_1} = \dfrac{a}{b} < 1 \rightarrow \varphi_0 < \varphi_1$

10) si $h > 0$ $\quad \dfrac{b}{a}$ grand $\quad b \gg a \quad \dfrac{a}{b} = \varepsilon$

$$\frac{\varphi_0}{\varphi_1} = \varepsilon + a\frac{h}{\lambda} \ln(1/\varepsilon)$$



$\frac{\varphi_0}{\varphi_1}$ augmente comme $\ln\left(\frac{b}{a}\right)$ donc ~~assez~~ lent

$\Rightarrow$ pas efficace.

iii) du coup, on prend $e = b - a$ faible

$b = a + e$ $\qquad \frac{b}{a} = 1 + \frac{e}{a} \to \varepsilon$

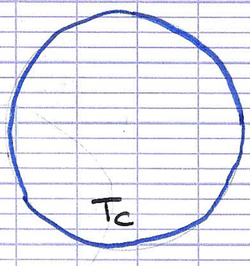
$\frac{\varphi_0}{\varphi_1} = \frac{1}{1+\varepsilon} + \frac{ah}{\lambda} \ln(1+\varepsilon)$ $\qquad$ il faut $\varphi_0/\varphi_1 > 1$

$= 1 - \varepsilon + \frac{ah}{\lambda}\varepsilon$ $\qquad \Rightarrow \frac{ah}{\lambda} > 1$

$= 1 + \varepsilon\left(\frac{ah}{\lambda} - 1\right)$ $\qquad$ il faut $\lambda < ah$

$h > \frac{\lambda}{a}$

Exercice 19



1) $\Delta T = 0$ en sta

2) $T(r, \theta, \varphi)$ invariance de rotation θ, φ $\Rightarrow T(r)$ + $T = cte$ car bon conducteur

3) A $T(r) = 0 \to \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$

$\Rightarrow r^2 \frac{dT}{dr} = cte = A'$

$\Rightarrow \dfrac{dT}{dr} = \dfrac{A'}{r} \Rightarrow T(r) = \dfrac{A}{r} + B$

4) Conditions limites

$T(r=a) = T_c$

$T(r \to +\infty) = T_f$

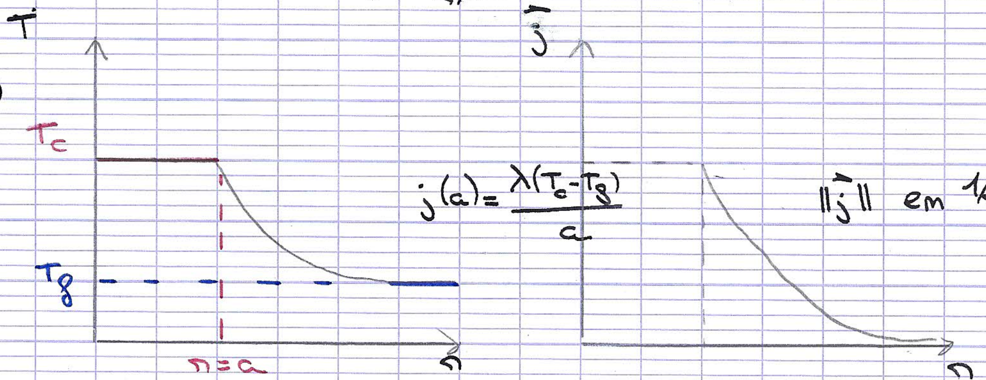
$\begin{cases} A/a + B = T_c \\ 0 + B = T_f \end{cases} \Rightarrow \begin{cases} A = T_c - T_F \\ B = T_F \end{cases}$

$T(r) = (T_c - T_f)\dfrac{a}{r} + T_f$

5) $\vec{j} = -\lambda \vec{\nabla} T = -\lambda \dfrac{dT}{dr}\vec{e_r} = -\lambda\,(T_c - T_f) \times \left(\dfrac{-a}{r^2}\right)\vec{e_r}$

$\vec{j} = \lambda a (T_c - T_f)\dfrac{1}{r^2}\vec{e_r}$ j en $1/r^2$

6)



7) $\vec{\varphi_0} = \iint \vec{j}\,\vec{dS}$ $\vec{j} = \lambda(T_c - T_f)\, a\, \dfrac{1}{r^2}\vec{e_r}$

$dV = r^2 \sin\theta\, d\theta\, dr\, d\varphi$

$\varphi(r=a) = \iint \dfrac{\lambda a (T_c - T_f)}{a^2}\, a^2 \sin\theta\, d\theta\, d\varphi$

$\varphi(r=a) = \lambda\, a\,(T_c - T_f)\left(\int_0^{2\pi} d\varphi\right) \times \left(\int_0^{\pi} \sin\theta\, d\theta\right)$

$= \lambda a\,(T_c - T_f)\, 4\pi$

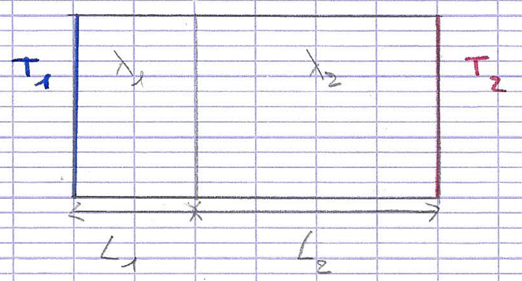
$\varphi_0 = 4\pi\lambda a\,(T_c - T_f)$ $\varphi = j \cdot S$

8) $T_c - T_f = R_{th}\,\varphi$ $R_{th} = \dfrac{1}{4\pi\lambda a}$

9) $R_{th} = \dfrac{1}{4\pi\ 0{,}6.10^{-3}} = 0{,}132.10^{8}\ W/K$

$P_c = \dfrac{T_c - T_8}{R_{Th}} \simeq 5{,}3\ \mu W$

Exercice 20



1) $\vec{j} = -\lambda \vec{\nabla} T$

$\dfrac{dT}{dt} = K \Delta T$

$\rightarrow$ stat donc $\Delta T = 0$

2) $\vec{j_1} = -\lambda_1 \dfrac{dT_1}{dx} \vec{e_x}$

$\dfrac{d^2T(x)}{dt^2} = 0$

$\vec{j_2} = -\lambda_2 \dfrac{dT_2}{dx} \vec{e_x}$

$\begin{cases} T_1(x) = a_1 x + b_1 \\ T_2(x) = a_2 x + b_2 \end{cases} \Rightarrow \begin{cases} T_1(x) = a_1 x + T_0 \\ T_2(x) = a_2 x + T_0 \end{cases}$

$b_1 = b_2 = T_0$

$\Downarrow$

$\begin{cases} T_1(x) = \dfrac{T_0 - T_1}{L_1} x + T_0 \\ T_2(x) = \dfrac{T_2 - T_0}{L_2} x + T_0 \end{cases}$

Conditions limites

$T_1(-L_1) = T_1$

$T_2(+L_2) = T_2$

$T_1(0) = T_0$

$T_2(0) = T_0$

$j_1(0) = j_2(0)$

3) $\vec{j}_1 = -\lambda_1 \frac{dT_1}{dx} \vec{e}_x$

$\vec{j}_1 = -\lambda_1 \frac{T_0 - T_1}{L_1} \vec{e}_x$ idem $\vec{j}_2 = -\lambda_2 \frac{T_2 - T_0}{L_2} \vec{e}_x$

$\Rightarrow T_0 = \frac{\lambda_1/L_1 \, T_1 + \lambda_2/L_2 \, T_2}{\lambda_1/L_1 + \lambda_2/L_2}$

4) $G_1 = \frac{\lambda_1}{L_1 S} = \frac{1}{R_1}$

$G_2 = \frac{\lambda_2}{L_2 S} = \frac{1}{R_2}$

$T_0 = \frac{G_1 T_1 + G_2 T_2}{G_1 + G_2}$

5) $L_1 = L_2$

$T_0 = \frac{\lambda_1 T_1 + \lambda_2 T_2}{\lambda_1 + \lambda_2}$

6) cas bois

$\lambda_1 = \lambda_{peau} = 10 \, W.m^{-1}.K^{-1}$

$\lambda_2 = \lambda_{bois} = 1 \, W.m^{-1}.K^{-1}$

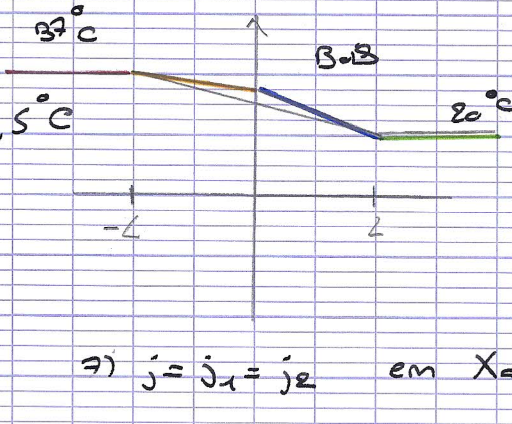
$T_0^{bois} = \frac{10 \times 37 + 1 \times 20}{11} \simeq 35{,}5 \, °C$

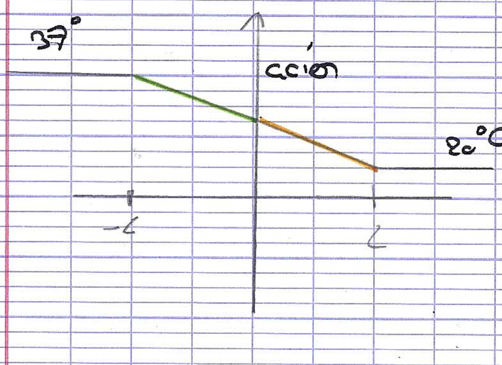
cas acier

$\lambda_1 = \lambda_{peau}$

$\lambda_2 = \lambda_{acier} = 100 \, W.m^{-1}.K^{-1}$

$T_0 = \frac{10 \times 37 + 100 \times 20}{110} \simeq 21{,}5 \, °C$



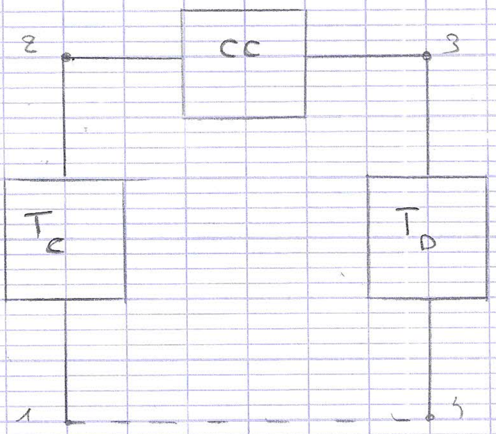


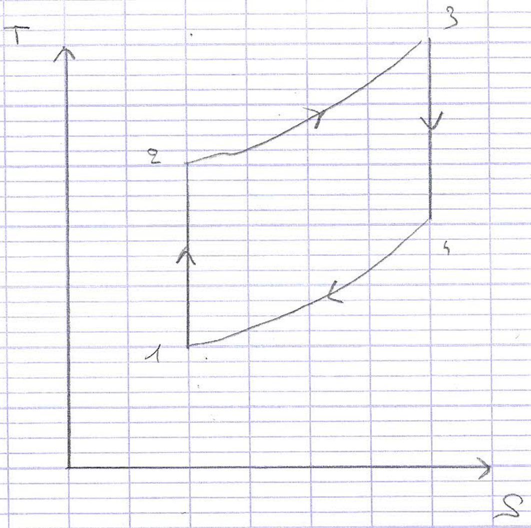
7) $j = j_1 = j_2$ em $x = 0$

$j = \lambda_1 \frac{T_1 - T_0}{L}$

$j = \frac{\lambda_1}{L}\left(T_1 - \frac{T_1 - T_0}{L}\right)$

Exercice 15





1 → 2 iso S

2 → 3 iso P réversible

3 → 4 iso P

4 → 1 iso P

1)

	1	2	3	4
P	1	6,5	6,5	1
T	300	T_2	1300	T_4
s	0	0	s^+	s^+

$s_2 = s_1 = s^- = 0 \ J.K^{-1}.kg^{-1}$

$s_3 = s_4 = s^+ = s_1 + c_p \ln\left(\frac{T_3}{T_4}\right) - \frac{R}{M} \ln\left(\frac{P_3}{P_4}\right)$

$= s_2 + c_p \ln\left(T_3/T_2\right) - \frac{R}{M} \ln\left(\frac{P_3}{P_1}\right)$

$\rightarrow \quad s_3 = s_2 + c_p \ln\left(\frac{T_3}{T_2}\right) - \frac{R}{M} \ln\left(\frac{P_3}{P_2}\right)$

$s_3 = c_p \ln\left(\frac{T_3}{T_2}\right) = 931,5 \ J.K^{-1}.kg^{-1}$

2) $\Delta(h + e_p + e_c) = w + q$

$\Delta h + \Delta e_p + \Delta e_c = w + q$ ici $\Delta h = w + q$

1 → 2 $\quad \Delta h_{12} = h_2 - h_1 = w_{12} + q_{12}$ 0 car adiabatique

⇒ $w_{12} = \Delta h_{12}$

gp → $T_1 \to T_2$

$w_{12} = c_p(T_2 - T_1) = 212 \text{ kJ.kg}^{-1}$

$w_{34} = c_p(T_4 - T_3) = -539 \text{ kJ.kg}^{-1}$

$w_{tot} = w_{12} + w_{34} = -327 \text{ kJ.kg}^{-1}$

< 0 donc moteur.

3) $\Delta h_{23} = w_{23} + q_{23}$

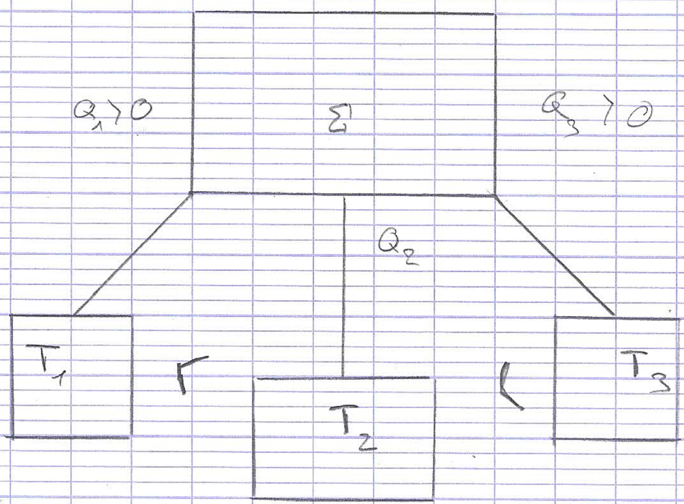
↳ $q_{23} = c_p(T_3 - T_2) = 788 \text{ kJ.kg}^{-1}$

4) $e = \dfrac{u}{d} = \dfrac{-w_{tot}}{q_c} \approx 0{,}41$

~~⇒ $w_{34} = c_p(T_4 - T_3) = -539$ kJ~~

~~$w_{tot} = w_{12} + w_{34} = -327$~~

Exercice 13



$Q_1 > 0$ car but du frigo

2) cycle ⇒ $\Delta U = 0$ ⇒ $W + Q_1 + Q_2 + Q_3 = 0$

$Q_1 + Q_2 + Q_3 = 0$

$\sum \dfrac{Q_i}{T_i} \leq 0$ ⇒ $\dfrac{Q_1}{T_1} + \dfrac{Q_2}{T_2} + \dfrac{Q_3}{T_3} \leq 0$

$$\Rightarrow -\frac{Q_3}{T_3} \geq \frac{Q_1}{T_1} + \frac{Q_2}{T_2} \quad \text{or} \quad Q_2 = Q_3 - Q_1$$

$$\Rightarrow -\frac{Q_3}{T_3} \geq \frac{Q_1}{T_1} - \frac{Q_3}{T_2} - \frac{Q_2}{T_2}$$

$$Q_3\left(\frac{1}{T_2} - \frac{1}{T_3}\right) \geq Q_1\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$Q_3 \geq Q_1 \frac{\frac{1}{T_1} - \frac{1}{T_2}}{\frac{1}{T_2} - \frac{1}{T_3}}$$

3) $e = \frac{u}{d} = \frac{Q_1}{Q_3}$

$$e \leq e_{max} = \frac{\frac{1}{T_2} - \frac{1}{T_3}}{\frac{1}{T_1} - \frac{1}{T_2}}$$

4) $e_c = \frac{T_g}{T_c - T_g} = \frac{1/T_c}{1/T_g - 1/T_c} = 18,5$

alors $e_{max} = \frac{1/T_c}{1/T_g - 1/T_c} - \frac{1/T_3}{1/T_g - 1/T_c} \leq e_c$
$= 14,0$

$$e_{max} \leq e_c$$