

# Mécanique du point

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## I - Cinématique du point

### Exercice 1: Un conducteur prudent

1) a)  $a = \frac{dv}{dt} \Leftrightarrow v = \int a \cdot dt = a \cdot t$

AN:  $24 = 2 \cdot t \Leftrightarrow t = \underline{12 \text{ s}}$

b)  $d = \int v \cdot dt = \frac{1}{2} a t^2$

AN:  $d = \frac{1}{2} \cdot 2 \cdot 12^2 = \underline{144 \text{ m}}$

2)  $\begin{cases} \text{En jipe: } d = v \cdot t \Rightarrow d = \underline{288 \text{ m}} \\ \text{En insertion: } d = \underline{144 \text{ m}} \end{cases} \text{ Pour } t = 12 \text{ s}$

IP saut donc  $288 - 144 + 20 = \underline{164 \text{ m}}$  de distance

### Exercice 2: Mouvement circulaire uniforme

1) Base polaire:

$$\begin{aligned} \vec{v} &= \dot{r} \vec{e}_r + r \dot{\varphi} \vec{e}_\varphi = R \omega \\ \vec{a} &= \ddot{r} \vec{e}_r + \dot{r} \dot{\vec{e}}_r + (r \ddot{\varphi} + \dot{\varphi} \dot{r}) \vec{e}_\varphi + r \dot{\varphi} \dot{\vec{e}}_\varphi \\ &= (\ddot{r} - r \dot{\varphi}^2) \vec{e}_r + (r \ddot{\varphi} + 2 \dot{r} \dot{\varphi}) \vec{e}_\varphi \\ &= -R \omega^2 \vec{e}_r \end{aligned}$$

Dans le repère de Frenet :  $\vec{a} = \frac{dv}{dt} \vec{t} + \frac{v^2}{R} \vec{n}$

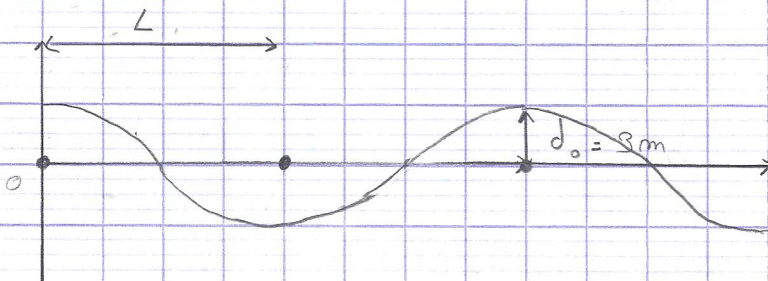
2) a)  $\omega = \frac{\text{mbr} \text{ tour} \times 2\pi}{\Delta t} = 6\pi$

$v = \underline{6,60 \text{ m} \cdot \text{s}^{-1}}$  ,  $a = \underline{124,36 \text{ m} \cdot \text{s}^{-2}}$



b)  $\omega = \int \dot{\omega} dt$       AN:  $\frac{6\pi}{5} = \dot{\omega}$   
 $\frac{d\omega}{dt} = \dot{\omega}$

Exercice 3 Test de stabilité d'une voiture



1) Équation de la forme  $y = A \cos Bx$

$$y(x=0) = A = d_0$$

$$y(x=L) = A \cos BL = -d_0$$

$$\Leftrightarrow \cos BL = -1$$

$$\Leftrightarrow BL = \pi$$

donc  $y = d_0 \cos\left(\frac{\pi}{L} x\right)$

2)  $v_y = \frac{dy(x)}{dt} = \frac{dy(x)}{dx} \frac{dx}{dt} = \left(-\frac{\pi}{L} d_0 \sin\left(\frac{\pi}{L} x\right)\right) \dot{x}$

$$v_x = \dot{x} = v_0$$

3)  $a_y = \frac{dv_y(x)}{dt} = \frac{dv_y(x)}{dx} \frac{dx}{dt} =$

$$-\frac{\pi^2}{L^2} d_0 \cos\left(\frac{\pi}{L} x\right) v_0^2$$

$$a_x = 0$$

4)  $\|a_y\| = \frac{\pi^2}{L^2} d_0 \left| \cos\left(\frac{\pi}{L} x\right) \right| v_0^2$

L'accélération est maximale en  $0 [L]$



5) IP faut que  $\|\vec{a}_{\max}\| \leq 0,7g$

$$0,87 > \frac{\pi^2}{L^2} d_0 \left| \cos \frac{\pi}{L} \right| v_0^2$$

$$0,87 > \frac{\pi^2}{L^2} d_0 v_0^2$$

$$\frac{\pi^2 d_0 v_0^2}{0,87} < L^2$$

$$L > \underline{28,8 \text{ m}}$$

6) Le mouvement n'est pas uniforme car  $\vec{v} \neq \text{cte}$

#### Exercice 4: Mouvement hélicoïdal

$$\begin{aligned} 1) \quad \vec{OM} &= x\vec{e}_x + y\vec{e}_y + z\vec{e}_z \\ &= r\cos\varphi\vec{e}_x + r\sin\varphi\vec{e}_y + h\vec{e}_z \end{aligned}$$

$$\vec{v} = \frac{d\vec{OM}}{d\varphi} \frac{d\varphi}{dt} + \frac{d\vec{OM}}{dz} \frac{dz}{dt}$$

$$\begin{aligned} &= (\cos\varphi\vec{e}_x + \sin\varphi\vec{e}_y)\dot{\varphi} + (-r\sin\varphi\vec{e}_x + r\cos\varphi\vec{e}_y + h\vec{e}_z)\dot{\varphi} \\ &= \dot{\varphi}\vec{e}_\varphi + r\dot{\varphi}\vec{e}_\varphi + h\dot{\varphi}\vec{e}_z \\ &= \omega(r\vec{e}_\varphi + h\vec{e}_z) \end{aligned}$$

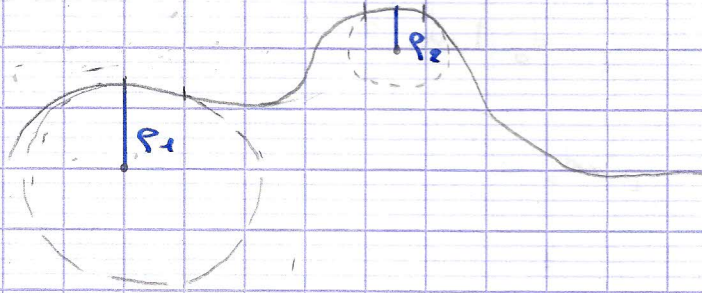
$$\|\vec{v}\| = \omega\sqrt{r^2 + h^2}$$

$$\begin{aligned} \vec{a} &= (\dot{\omega}r + \omega\dot{r})\vec{e}_\varphi - \omega^2 r\vec{e}_\rho + (\dot{\omega}h + \omega\dot{h})\vec{e}_z \\ &= \dot{\omega}r\vec{e}_\varphi - \omega^2 r\vec{e}_\rho + \dot{\omega}h\vec{e}_z \end{aligned}$$

$$\|\vec{a}\| = \sqrt{\dot{\omega}^2 r^2 + \omega^4 r^2 + \dot{\omega}^2 h^2}$$



Rayon de courbure:



$\rho$  : Rayon de courbure

$$\omega = cfe$$

$$\dot{\omega} = 0$$

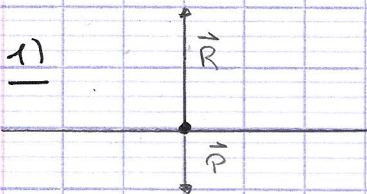
$$r_1 = \frac{\omega \cdot \pi}{R} + \frac{v^2}{R}$$

$$= \frac{\omega^2 (r^2 + h^2)}{R}$$



## II - Dynamique du point

### Exercice 5: Forces exercées par des sportifs



$$\text{PFD: } \Sigma \vec{F} = m \cdot \vec{a}$$

$$\vec{P} + \vec{R} = m \cdot \vec{a}$$

$$\text{or } \vec{P} = -3g$$

$$\vec{P} = mg$$

$$\text{d'où } \vec{R} = -4m \cdot \vec{g}$$

2)

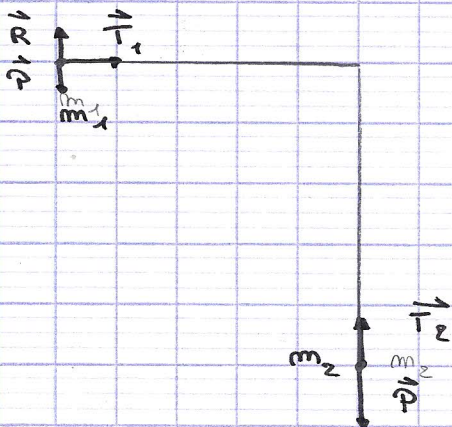
$$v^2 - v_0^2 = 2a_0(x - x_0)$$

$$v^2 = 2a_0 x$$

$$a_0 = \frac{v^2}{2x} \Rightarrow a_0 = \underline{400 \text{ m} \cdot \text{s}^{-2}}$$

$$\vec{F} = \vec{P} = m \cdot \vec{a} = \underline{68 \text{ N}}$$

### Exercice 6: Mouvement de deux blocs



Système 1:

$$\vec{P} + \vec{R} + \vec{T} = m \cdot \vec{a}$$

$$m \cdot g \cdot \vec{e}_y - R \vec{e}_y + T_1 \vec{e}_x = m_1 a_1 \vec{e}_x$$

$$\begin{cases} R = m \cdot g \\ T_1 = m_1 a_1 \end{cases}$$

Système 2

$$\vec{P} + \vec{T} = m \cdot \vec{a}$$

$$m \cdot g \vec{e}_z - T_2 \vec{e}_z = m \cdot a \vec{e}_z$$

$$m_2 a_2 = m_2 g - T_2$$

$$\text{or } a_1 = a_2 = a$$

$$T_1 = T_2 = T$$



donc

$$m_1 a = m_2 g - m_2 a$$

$$a = \frac{m_2 g}{m_1 + m_2}$$

$$= \underline{3,27 \text{ m.s}^{-2}}$$

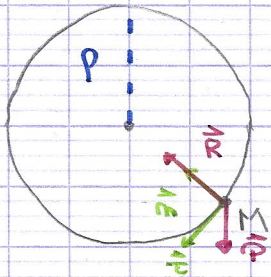
$$T = \underline{65,4 \text{ N}}$$

2)  $x = \int \int a dt$

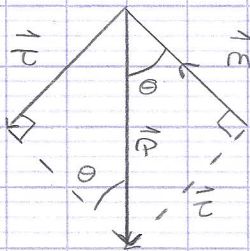
$$= \frac{1}{2} a t^2$$

$$= \underline{6,54 \text{ m}} \quad \text{pour } t = 2 \text{ s}$$

### Exercice 7: le guide circulaire



a) PFD:  $\vec{P} + \vec{R} = m \cdot \vec{a}$



$$\vec{P} = 2 \sin \theta mg \vec{e}_r - \cos \theta mg \vec{e}_t$$

$$\vec{R} = R \vec{e}_r$$

$$2 \sin \theta mg \vec{e}_r - \cos \theta mg \vec{e}_t + R \vec{e}_r = m \left( \frac{dv}{dt} \vec{e}_t + \frac{v^2}{\rho} \vec{e}_r \right)$$

donc

$$\begin{cases} 2 \sin \theta mg = m \frac{dv}{dt} & (r) \\ R - \cos \theta mg = m \frac{v^2}{\rho} & (n) \end{cases} \quad \text{or } v = \rho \dot{\theta}$$

$$\begin{cases} \rho \ddot{\theta} = 2 \sin \theta g \\ \rho \dot{\theta}^2 = R - \cos \theta mg \end{cases}$$

b) TMC:  $\Sigma M_o = \frac{d\sigma_o}{dt}$

$$M_o(R) = \vec{OM} \wedge \vec{R} = -\rho \vec{e}_r \wedge R \vec{e}_r = \vec{0}$$

$$M_o(P) = \vec{OM} \wedge \vec{P} = -\rho \vec{e}_r \wedge (2 \sin \theta mg \vec{e}_r - \cos \theta mg \vec{e}_t) = \rho \sin \theta mg \vec{b}$$

$$\sigma_o = \vec{OM} \wedge m \vec{v} = -\rho \vec{e}_r \wedge m v \vec{e}_t = \rho^2 m \dot{\theta} \vec{b}$$

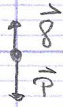


$$M_0(\ddot{\vec{R}}) + M_0(\ddot{\vec{P}}) = P^e m \ddot{\vec{b}}$$

$$2 \sin \theta g P_m = \ddot{P}^e m$$

$$P \ddot{\theta} = 2 \sin \theta g$$

### Exercice 8: Mouvement vertical d'une goutte d'eau



1) a)

PFD:  $\Sigma \vec{F} = m \cdot \vec{a}$

$$m_0 g \vec{e}_z - \alpha m_0 v \vec{e}_z = m_0 \cdot a \vec{e}_z$$

$$= m_0 \cdot \frac{dv}{dt} \vec{e}_z$$

donc  $\dot{v} = g - \alpha v$

$v = v_p$  quand  $a = \dot{v} = 0$

$$g - \alpha v_p = 0$$

$$\alpha v_p = g$$

$$v_p = \frac{g}{\alpha}$$

$$v_p = \frac{1}{4} \text{ m.s}^{-1}$$

b)  $y' = g - \alpha y$   
 $y' + \alpha y = g$

$$y_H = C \cdot e^{-\alpha x}$$

$$y_0 = e^{\alpha x}$$

$$C(\alpha) = \frac{g}{\alpha} e^{\alpha x}$$

$$y_p = \frac{g}{\alpha}$$

$$v = v_p + e^{-\alpha t} \cdot (-v_p)$$

à  $t=0, v=0$  donc

$$C = -v_p$$

c) On cherche  $\frac{v_p - v(T)}{v_p} = \frac{1}{1000}$

$$v(T) = v_p - \frac{v_p}{1000}$$

$$\frac{v_p}{1000} + e^{-\alpha t} \cdot (-v_p) = v_p - \frac{v_p}{1000}$$

$$-\alpha t = P_m \left( \frac{1}{1000} \right), \quad t = \frac{P_m (1/1000)}{-\alpha}$$

$$t = 0,173$$



$$2) \quad \rho = \frac{m}{\frac{4}{3}\pi R^3} \Rightarrow \frac{4}{3}\pi R^3 \rho = m$$

$$a) \quad \frac{dm}{dt} = \frac{dm}{dR} \frac{dR}{dt} = \underline{4\pi R^2 \rho \cdot R_0 b}$$

$$b) \quad \text{PFD: } \Sigma \vec{F} = \frac{dm}{dt} \vec{v} + m \frac{d\vec{v}}{dt}$$

$$m \cdot g = (4\pi R^2 \rho R_0 b) v + m \dot{v}$$

$$g = \frac{3 R_0 b}{R} v + \dot{v}$$

$$c) \quad \dot{v} + \frac{3 R_0 b}{R} v = g \quad \Leftrightarrow \quad \dot{v} + \frac{3}{R} v = \frac{g}{R_0 b}$$

$$v_H = C \cdot e^{\int -\frac{3}{R} dR} = C \cdot R^{-3}$$

$$v_0 = R$$

$$C'(R) = \frac{g}{R_0 b} R^3$$

$$C(R) = \frac{g R^4}{4 R_0 b}$$

$$v_p = \frac{g R}{4 R_0 b}$$

$$v = C \cdot R^{-3} + \frac{g R}{4 R_0 b}$$

$$\text{à } t=0, v=v_p,$$

$$R=R_0 \text{ donc}$$

$$v_p = C \cdot R_0^{-3} + \frac{g}{4b}$$

$$\text{donc } C = \left(v_p - \frac{g}{4b}\right) R_0^3$$

$$\text{Ainsi: } v = \left(v_p - \frac{g}{4b}\right) R_0^3 R^{-3} + \frac{g R}{4b R_0}$$

$$v(t) = \left(v_p - \frac{g}{4b}\right) (1+bt)^3 + \frac{g(1+bt)}{4b}$$

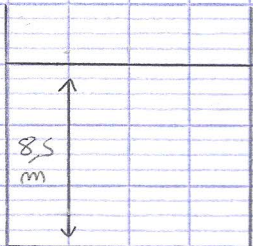
$$d) \quad a = \frac{dv}{dt} = \frac{-3b \left(v_p - \frac{g}{4b}\right)}{(1+bt)^4} + \frac{g}{4}$$

$$\text{Pim } a = \frac{g}{4}$$



### III - Travail et énergie

#### Exercice 9 : Intérêt des uzima macromatrics



$$\begin{aligned}\delta W &= \vec{g} \cdot d\vec{R} = \vec{g} \cdot \vec{v} \cdot dt \\ &= -d\mathcal{E}_p\end{aligned}$$

On décompose en tranches élémentaire de hauteur  $z$  et d'épaisseur  $dz$ , de masse  $dm$

$$d\mathcal{E}_p = dm \cdot g \cdot z$$

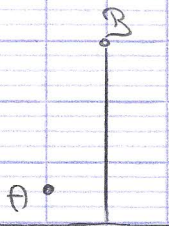
$$\delta W = -dm \cdot g \cdot z$$

$$= -\rho \cdot S \cdot g \cdot z \cdot dz$$

$$W = -\frac{1}{2} \rho \cdot S \cdot g \cdot z^2$$

$$\text{AN: } W = \left[ -\frac{1}{2} \rho \cdot S \cdot g \cdot z^2 \right]_{8,5}^0 = \underline{\underline{8,15 \cdot 10^{12} \text{ J}}}$$

#### Exercice 10 : Saut à la perche



$$\begin{aligned}\mathcal{E}_m(A) &= \mathcal{E}_c(A) + \mathcal{E}_p(B) \\ &= \frac{1}{2} m v^2 + m g z\end{aligned} \quad (1)$$

$$\mathcal{E}_m(B) = m g h \quad (2)$$

La seule force étant conservative il y a conservation de l'énergie mécanique donc (1) = (2)

$$\frac{1}{2} m v^2 + m g z = m g h$$

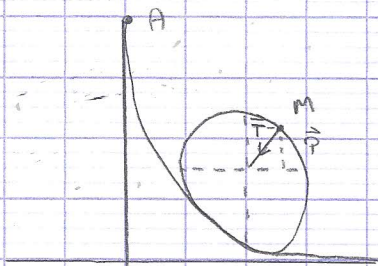
$$h = \frac{1}{2g} v^2 + z$$

$$h = \underline{\underline{6,09 \text{ m}}}$$



## Exercice 11: Le Looping

Repère de Frenet



$$\text{PFD}(M): \Sigma \vec{F} = m \cdot \vec{a}$$

$$\vec{T} + \vec{P} = m \cdot \vec{a}$$

$$T \cdot \vec{n} + mg(-\sin \theta \vec{n} + \cos \theta \vec{t}) =$$

$$m \left( \frac{dv}{dt} \vec{t} + \frac{v^2}{R} \vec{n} \right)$$

$$\begin{cases} T + \sin \theta mg = \frac{v^2}{R} m & (m) \\ \cos \theta g = \frac{dv}{dt} & (v) \end{cases}$$

$$\text{donc } T = \frac{v^2}{R} m - \sin \theta mg \quad \text{avec } \vec{z}$$

Il y a conservation de l'énergie mécanique,  $P$  est une force conservative,  $T$  est perpendiculaire.

$$E_m(A) = E_m(M)$$

$$mgh = \frac{1}{2} mv^2 + mgz \quad \text{avec } z = R + R \sin \theta$$

$$v^2 = 2g(h - z)$$

$$v^2 = 2g(h - R - R \sin \theta)$$

$$\text{donc } T = \frac{2g(h - R - R \sin \theta)}{R} m - \sin \theta mg$$

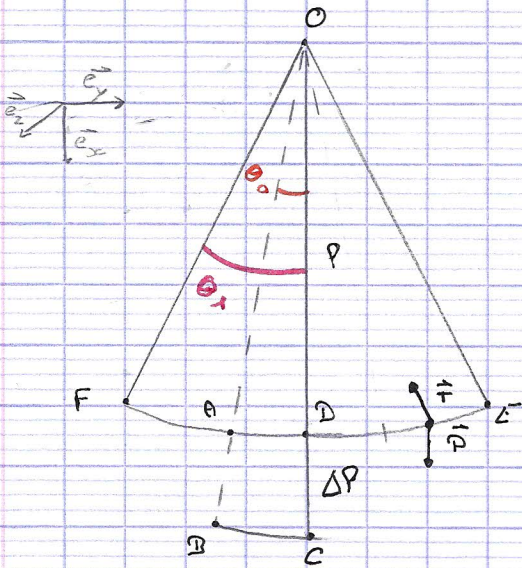
$$= mg \left( \frac{2h}{R} - 2 - \sin \theta \right)$$

$$\text{en } \theta = \frac{\pi}{2} : T_m = mg \left( \frac{2h}{R} - 5 \right) \geq 0$$

$$h \geq \frac{5}{2} R$$



## Exercice 12: Mouvement de Pa balancière



$$\begin{aligned} 1) \quad \vec{L}_O^m(C) &= \vec{r}_{OC} \wedge m\vec{v} \\ \vec{L}_O^m(C) &= OC \wedge m\vec{v} \\ &= (P + \Delta P) \vec{e}_x \wedge m v_c \vec{e}_y \\ &= \underline{(P + \Delta P) m v_c} \underline{\vec{e}_z} \end{aligned}$$

$$\begin{aligned} \vec{L}_O^m(D) &= \vec{OD} \wedge m\vec{v} \\ &= P \vec{e}_x \wedge m v_D \vec{e}_y \\ &= \underline{P m v_D} \underline{\vec{e}_z} \end{aligned}$$

donc  $v_D P m = (P + \Delta P) m v_c$

$$v_D = \frac{P + \Delta P}{P} v_c > v_c \quad \frac{P + \Delta P}{P} > 1$$

Ainsi, P gagne de la vitesse

$$\begin{aligned} 2) \quad \vec{L}_m^m(A) &= \vec{L}_m^m(C) \\ m g z_A &= \frac{1}{2} m v^2 \quad \text{avec } z_A = R - R \cos \theta \\ v^2 &= 2g(R - R \cos \theta) \\ v_c^2 &= 2g(P + \Delta P)(1 - \cos \theta_0) \\ v_D^2 &= 2g P (1 - \cos \theta_1) = \left(\frac{P + \Delta P}{P}\right)^2 v_c^2 \end{aligned}$$

$$\begin{aligned} 2g P (1 - \cos \theta_1) &= \left(\frac{P + \Delta P}{P}\right)^2 2g (P + \Delta P) (1 - \cos \theta_0) \\ \frac{\sin \theta_1/2}{\sin \theta_0/2} &= \left(\frac{P + \Delta P}{P}\right)^{3/2} \end{aligned}$$

$$\begin{aligned} 3) \quad \theta_0 &= \frac{\pi}{2}, \quad \sin \theta \xrightarrow{\theta \rightarrow 0} \theta, \quad \sin \frac{\theta_0}{2} = \frac{\pi}{4} \\ \sin \frac{\theta_1}{2} &= \sin \frac{\theta_0}{2} \cdot R = R \frac{\pi}{4} \\ \sin \frac{\theta_2}{2} &= R^2 \frac{\pi}{4} \\ \sin \frac{\theta_n}{2} &= R^n \frac{\pi}{4} \end{aligned}$$

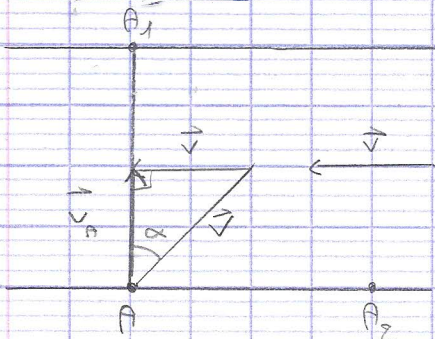
Horizontal si  $\theta_n \geq \frac{\pi}{2} \Leftrightarrow \frac{\theta_n}{2} \geq \frac{\pi}{4} \Leftrightarrow \sin \frac{\theta_n}{2} \geq \frac{\sqrt{2}}{2}$

$$n \geq \frac{\frac{1}{2} P m 2 - P m \pi}{P m R}$$



## IV - Changement de référentiel

### Exercice 13: Problème du mageur



(R) : la berge

(R') : l'eau

$\vec{v} = \vec{v}_e$  = vitesse de l'eau R/R

$\vec{V} = \vec{v}_R$  = vitesse mageur / eau / R'

$\vec{v}_a = \vec{v}_e + \vec{v}_R$  vitesse mageur / berge / R

1)

Trajet AA<sub>1</sub>A:

donnée  $t_1, t_1 = t_1' + t_1'' = 2t_1'$

$$t_1 = \frac{2d}{v_a}$$

$$v_a^2 = V^2 - v^2$$

pythagore

$$t_1 = \frac{2d}{\sqrt{V^2 - v^2}}$$

$$\frac{t_2}{t_1} > 1 \text{ donc } t_2 > t_1$$

Trajet AA<sub>2</sub>A:

$$t_2 = t_2' + t_2''$$

$$t_2' = \frac{d}{V-v}$$

$$t_2 = \frac{2dV}{V^2 - v^2}$$

$$t_2'' = \frac{d}{V+v}$$

2)  $t_2 = 2t_1 = 7 \text{ min}$

$\sin \alpha = \frac{v}{V}$

$$\frac{1}{\sqrt{1 - \frac{v^2}{V^2}}} = 2 \Rightarrow 1 - \frac{v^2}{V^2} = \frac{1}{4} \Rightarrow \frac{v}{V} = \frac{\sqrt{3}}{2} \Rightarrow \alpha = \frac{\pi}{3}$$



$$t_0 = \frac{2d}{v}, \quad t_z = \frac{2dv}{v^2 - v} = \frac{2d}{v} \frac{1}{\sqrt{1 - \frac{v^2}{v^2}}}$$

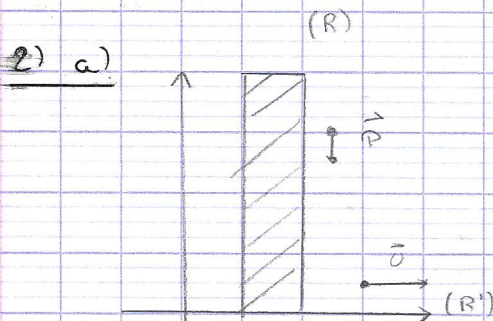
$$t_x = \frac{t_0}{\sqrt{1 - \frac{v^2}{v^2}}}$$

avec  $\frac{v^2}{v^2} = \frac{3}{4}$ ,  $t_x = \frac{t_0}{\sqrt{1 - \frac{3}{4}}} = 2t_0$

$$\begin{cases} t_0 = \frac{3}{4} \text{ min} \\ t_x = \frac{3}{2} \text{ min} \\ t_z = \frac{3}{2} \text{ min} \end{cases}$$

### Exercice 14: chute d'un objet dans différents référentiels

1) La masse va tomber en bas du mat



dans (R)

$$\vec{P} = m\vec{g} = m\vec{a} \Rightarrow \vec{a} = \vec{g}$$

$$\begin{cases} a_x = 0 \\ a_y = -g \end{cases} \Leftrightarrow \begin{cases} v_x = 0 \\ v_y = -gt + c_2 \end{cases}$$

à  $t=0$  la masse est au repos

donc  $v_x = v_y = 0$

$$c_1 = c_2 = 0$$

$$\begin{cases} x = d_1 \\ y = -\frac{1}{2}gt^2 + d_2 \end{cases}$$

à  $t=0$   $d_1 = 0$   
 $d_2 = h$

$$\rightarrow y = -\frac{1}{2}gt^2 + h$$

dans (R') référentiel galiléen on a encore  $\vec{a}' = \vec{g}$

$$\vec{v}_a = \vec{v}_R + \vec{v}_e \Leftrightarrow \vec{v} = \vec{v}' + \vec{u} \Leftrightarrow \vec{v}' = \vec{v} - \vec{u}$$

$$\begin{cases} v'_x = -u \\ v'_y = v_y = -gt \end{cases} \Leftrightarrow \begin{cases} a'_x = -\dot{u} + R_1 \\ y' = \frac{1}{2}gt^2 + R_2 \end{cases}$$

$$R_1 = 0, R_2 = h$$



b)

$$\begin{aligned} \vec{a}' &= \vec{a} - \dot{\vec{p}} \\ &= -g \vec{e}_y - a_e \vec{e}_x \end{aligned}$$

$$\vec{a}' \begin{cases} a'_x = -a_e \\ a'_y = -g \end{cases}$$

$$\Leftrightarrow \begin{cases} x' = -\frac{1}{2} g a_e t^2 \\ y' = -\frac{1}{2} g t^2 + h \end{cases}$$

$$\Rightarrow \begin{cases} \dot{x}' = -a_e t \\ \dot{y}' = -g t \end{cases}$$

$$y' = \frac{g}{a_e} x' + h$$

mouvement parabolique

### Exercice 16 Référentiel terrestre non galiléen (R)

$$\begin{aligned} \underline{1)} \quad \sum \vec{g} + \vec{g}'_{ie} + \vec{g}'_{ic} &= m \cdot \vec{a}'_R \\ \vec{g} - m \vec{a}_e - m \vec{a}_c &= m \cdot \vec{a}'_R \\ m \vec{g} - 2m \vec{\omega} \wedge \vec{v}'_R &= m \cdot \vec{a}'_R \\ \vec{g} - 2\vec{\omega} \wedge \vec{v}'_R &= \vec{a}'_R \end{aligned}$$

$$\begin{aligned} \underline{2)} \quad \vec{g} &= -g \vec{e}_z \\ \vec{\omega} &= \omega \cos \lambda \vec{e}_y + \omega \sin \lambda \vec{e}_z \\ \vec{v} &= \dot{x} \vec{e}_x + \dot{y} \vec{e}_y + \dot{z} \vec{e}_z \\ \vec{\omega} \wedge \vec{v} &= (\dot{z} \omega \cos \lambda - \dot{y} \omega \sin \lambda) \vec{e}_x + \dot{x} \omega \sin \lambda \vec{e}_y - \dot{x} \omega \cos \lambda \vec{e}_z \end{aligned}$$

$$\begin{aligned} \vec{a}'_x &= -2\omega (\cos \lambda \dot{z} - \dot{y} \sin \lambda) \\ \vec{a}'_y &= -2\omega \dot{x} \sin \lambda \\ \vec{a}'_z &= -g + 2\omega \dot{x} \cos \lambda \end{aligned}$$

$$\underline{3)} \quad \begin{aligned} \dot{y} &= -2\omega x \sin \lambda + c_1 t, \quad c_1 = 0 \\ \dot{z} &= -g t + 2\omega x \cos \lambda + c_2 t, \quad c_2 = 0 \end{aligned}$$

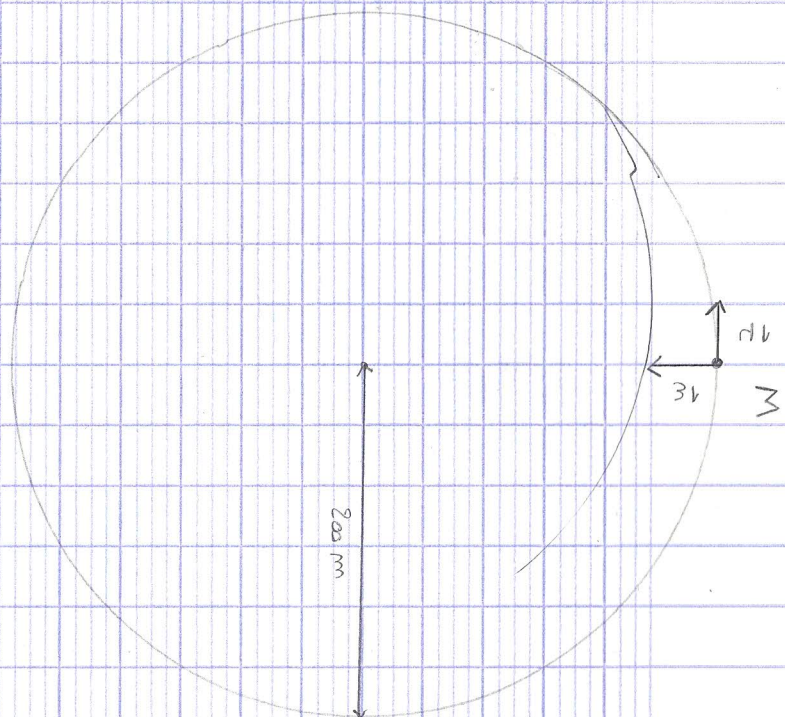
$$\underline{4)} \quad \begin{aligned} \ddot{x} &= -2\omega (-g t \cos \lambda + 2\omega x \cos \lambda + 2\omega x \sin^2 \lambda) \\ &= +2\omega g t \cos \lambda - 4\omega^2 x \end{aligned}$$



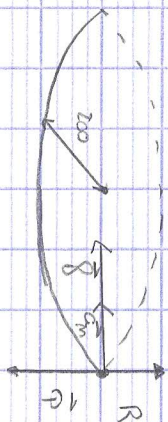




Plot



$$a_c = \frac{v}{R} \frac{dv}{dt}$$

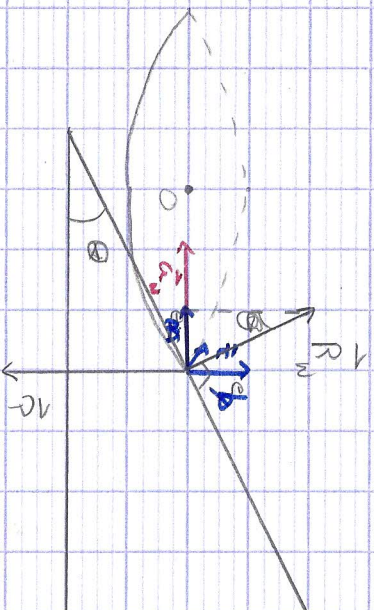


$$\sum F_{ext} = m \cdot a \Rightarrow \vec{R} + \vec{R}_N + \vec{g} = m \cdot a$$

$$\sum F_{ext} = m \cdot a_N \Rightarrow \vec{g} = m \cdot a_N \Rightarrow g = m \frac{v^2}{R}$$

Si  $m \approx 1000 \text{ kg}$ ,  $g \approx 9.80665 \text{ N}$

Relevé



$$\sum F = m \cdot a \Rightarrow \vec{R} + \vec{R}_N = m \cdot a$$

$$\int R_N \sin \theta = m \cdot \frac{v^2}{R} \quad (m)$$

$$\int R_N \cos \theta = mg \quad (h)$$

$$\Leftrightarrow v^2 = Rg \tan \theta$$

$$\Leftrightarrow \tan \theta = \frac{v^2}{Rg}$$

$$\Leftrightarrow \theta = 23,27^\circ$$