

Mécanique - Travaux dirigés

1

$$R(O, \vec{e}_1, \vec{e}_2, \vec{e}_3)$$

$$\vec{u}_1, \vec{u}_2, \vec{u}_3$$

$$1) \vec{u}_1 \wedge (\vec{u}_2 \wedge \vec{u}_3) = (\vec{u}_1 \cdot \vec{u}_3) \vec{u}_2 - (\vec{u}_1 \cdot \vec{u}_2) \vec{u}_3 \quad ? \text{ vrai}$$

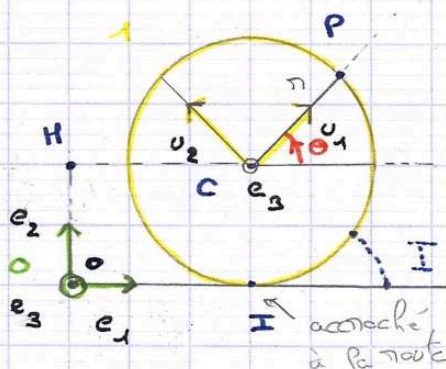
Soit $\vec{u}_1 (x_1, y_1, z_1)$, $\vec{u}_2 (x_2, y_2, z_2)$ et $\vec{u}_3 (x_3, y_3, z_3)$

$$2) \vec{u}_1 \wedge (\vec{u}_2 \wedge \vec{u}_3) = (\vec{u}_1 \wedge \vec{u}_2) \wedge \vec{u}_3 = -\vec{u}_3 \wedge (\vec{u}_1 \wedge \vec{u}_2)$$

$$= -[(\vec{u}_3 \cdot \vec{u}_2) \vec{u}_1 - (\vec{u}_3 \cdot \vec{u}_1) \vec{u}_2] = (\vec{u}_3 \cdot \vec{u}_1) \vec{u}_2 - (\vec{u}_3 \cdot \vec{u}_2) \vec{u}_1$$

$\Rightarrow$ faux.

Exercice : Cinématique 1



$P \in \mathcal{C}$ (méthode directe)

$$\vec{V}_{P \in \mathcal{C}} = \frac{d_0 \vec{OP}}{dt}$$

$$= \frac{d_0}{dt} (\vec{OH} + \vec{HC} + \vec{CP})$$

$$= \frac{d_0}{dt} (x \vec{e}_1 + r \vec{u}_1)$$

$$= \dot{x} \vec{e}_1 + r \frac{d_0 \vec{u}_1}{dt}$$

$$= \dot{x} \vec{e}_1 + r (\vec{\Omega}_{\mathcal{C}} \wedge \vec{u}_1)$$

$$= \dot{x} \vec{e}_1 + r (\dot{\theta} \vec{e}_3 \wedge \vec{u}_1)$$

$$= \dot{x} \vec{e}_1 + r \dot{\theta} \vec{u}_2$$

$$\vec{V}_{I \in \mathcal{C}} = \frac{d_0 \vec{OI}}{dt}$$

$$\vec{V}_{I \in \mathcal{C}} = \dot{x} \vec{e}_1 + r \dot{\theta} \vec{e}_1 = (\dot{x} + r \dot{\theta}) \vec{e}_1$$

Le point I est le point géométrique de vitesse

$$\vec{V} = \dot{x} \vec{e}_1$$

S'il y a roulement sans glissement au contact

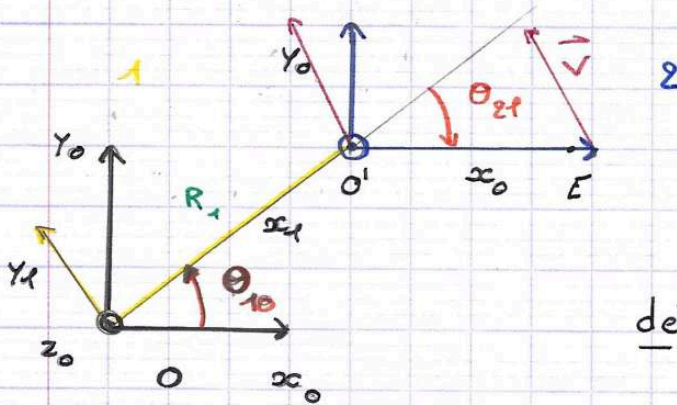
$$\Rightarrow \vec{V}_{I \in \mathcal{C}} = \vec{0} \Rightarrow \dot{x} = -r \dot{\theta} \Rightarrow x = -r \theta$$

Méthode alternative: (à privilégier)

$$\vec{V}_{C \in 10} = \frac{d_0 \vec{\alpha}}{dt} = \frac{d_0 \vec{HC}}{dt} = \frac{d_0 \alpha \vec{e}_x}{dt} = \dot{\alpha} \vec{e}_x$$

$$\begin{aligned} \Rightarrow \vec{V}_{P \in 10} &= \vec{V}_{C \in 10} + \vec{PC} \wedge \vec{\Omega}_{10} (\text{ou } \vec{\Omega}_{10} \wedge \vec{CP}) \\ &= \begin{cases} \dot{\alpha} \vec{e}_x + \dot{\theta} \vec{e}_3 \times \eta \vec{u} \\ \dot{\alpha} \vec{e}_x + \eta \dot{\theta} \vec{e}_2 \end{cases} \end{aligned}$$

Mécanique



déplacement parallèle

Mouvement 2/0 est absolue et c'est une translation circulaire.

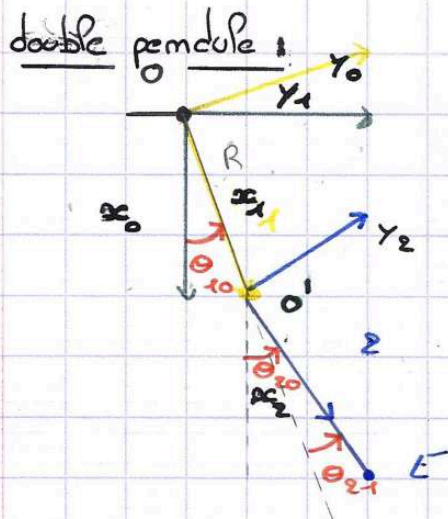
$$\begin{cases} \vec{V}_{20} \end{cases} = \begin{cases} \vec{\Omega}_{20} = \vec{0} \\ \vec{V}_{O' \in 2/0} = \end{cases} = \begin{cases} \vec{\Omega}_{21} = \dot{\theta}_{10} \vec{z}_0 \\ \vec{V}_{O' \in 2/1} = \vec{0} \end{cases} + \begin{cases} \vec{\Omega}_{10} = \dot{\theta}_{10} \vec{z}_0 \\ \vec{V}_{O' \in 1/0} = \vec{V}_{O \in 1/0} + \vec{\Omega}_{10} \wedge \vec{OO'} \end{cases}$$

$$\dot{\theta}_{21} + \dot{\theta}_{10} = 0 \Rightarrow \dot{\theta}_{21} = -\dot{\theta}_{10} \Rightarrow \theta_{21} = -\theta_{10} + cte$$

$$\text{Gm a } \vec{V}_{O' \in 1/0} = \vec{V}_{O \in 1/0} + \vec{\Omega}_{10} \wedge \vec{OO'} \\ = \dot{\theta}_{10} \vec{z}_0 \wedge R_1 \vec{x}_1 = R_1 \dot{\theta}_{10} \vec{y}_1$$

$$\begin{cases} \vec{V}_{20} \end{cases} = \begin{cases} \vec{\Omega}_{20} = \vec{0} \\ \vec{V}_{K \in 2/0} = \vec{V}_{O' \in 2/0} + \vec{\Omega}_{20} \wedge \vec{O'O} \\ = R_1 \dot{\theta}_{10} \vec{y}_1 \end{cases}$$

$$\rightarrow \begin{cases} \vec{\Omega}_{20} = \vec{0} \\ \vec{V}_{K \in 2/0} = R_1 \dot{\theta}_{10} \vec{y}_1 \quad \forall K \in 2 \end{cases}$$



quelle est l'accélération
 $\vec{a}_{C \in 210}$?

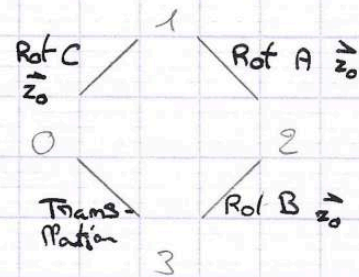
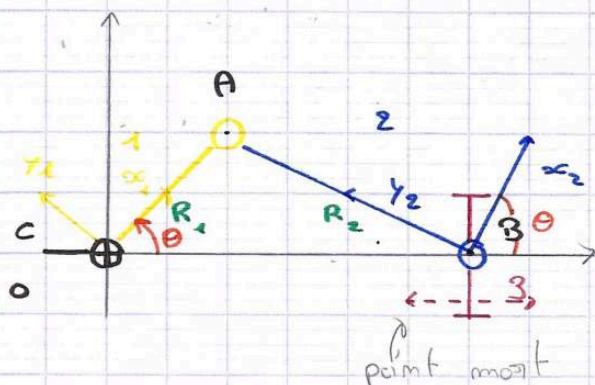
Cherchons $\vec{v}_{C \in 210}$ par composition et distribution sachant que $\vec{z}_0 = \vec{z}_1 + \vec{l}_0$
 abs rel ent
 $\downarrow \quad \downarrow$
 $cm O' \quad cm O \rightarrow cm O'$

$$\begin{aligned} \vec{v}_{C \in 210} &= \vec{v}_{C \in 211} + \vec{v}_{C \in 110} \\ \vec{v}_{C \in 211} &= \vec{v}_{O' \in 211} + \vec{\Omega}_{21} \wedge \vec{O'C} = \dot{\theta}_{21} \vec{z}_0 \wedge l_2 \vec{x}_2 \\ &= l_2 \dot{\theta}_{21} \vec{y}_2 \\ \vec{v}_{C \in 110} &= \vec{v}_{O \in 110} + \vec{\Omega}_{10} \wedge (\vec{OO'} + \vec{O'C}) \\ &= \dot{\theta}_{10} \vec{z}_0 \wedge (l_1 \vec{x}_1 + l_2 \vec{x}_2) \\ &= l_1 \dot{\theta}_{10} \vec{y}_1 + l_2 \dot{\theta}_{10} \vec{y}_2 \\ \text{d'où } \vec{v}_{C \in 210} &= l_1 \dot{\theta}_{10} \vec{y}_1 + l_2 (\dot{\theta}_{10} + \dot{\theta}_{21}) \vec{y}_2 \\ &= l_1 \dot{\theta}_{10} \vec{y}_1 + l_2 \dot{\theta}_{20} \vec{y}_2 \end{aligned}$$

On dérive la vitesse pour obtenir l'accélération

$$\begin{aligned} \vec{a}_{C \in 20} &= \frac{d_0 \vec{v}}{dt} \vec{e}_{20} \\ &= l_1 (\ddot{\theta}_{10} \vec{y}_1 + \dot{\theta}_{10} \frac{d_0 \vec{y}_1}{dt}) + l_2 (\ddot{\theta}_{20} \vec{y}_2 + \dot{\theta}_{20} \frac{d_0 \vec{y}_2}{dt}) \\ &= \vec{\Omega}_{20} \wedge \vec{y}_2 \quad \vec{\Omega}_{20} \wedge \vec{y}_2^i \\ &= l_1 (\ddot{\theta}_{10} \vec{y}_1 + \dot{\theta}_{10} (-\vec{x}_1)) + l_2 (\ddot{\theta}_{20} \vec{y}_2 + \dot{\theta}_{20} (-\vec{x}_2)) \end{aligned}$$

La bielle:



On a une géométrie géométrique: $\vec{O} = \vec{CC} = \vec{CA} + \vec{AB} + \vec{BC}$
 $= R\vec{x}_1 - L\vec{y}_2 - x\vec{x}_0$

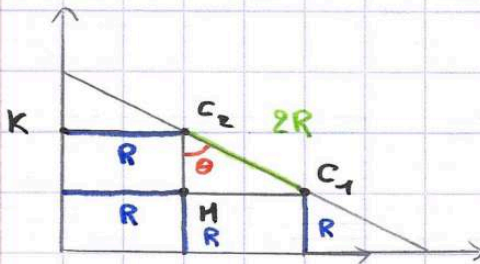
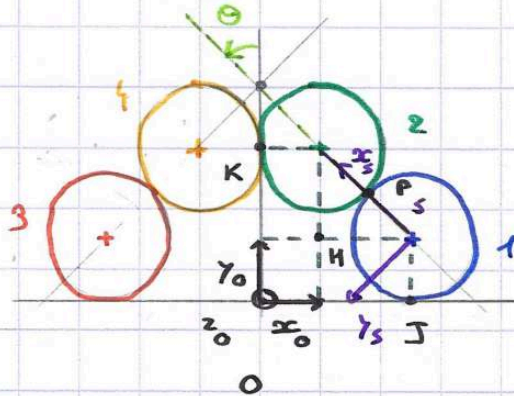
en $\vec{x}_0$ $\vec{O} = R\vec{x}_1\vec{x}_0 - L\vec{y}_2\vec{x}_0 - x\vec{x}_0\vec{x}_0$
 $= R\cos\theta + L\sin\beta - x$

en $\vec{y}_0$ $\vec{O} = R\vec{x}_1\vec{y}_0 - L\vec{y}_2\vec{y}_0 - x\vec{x}_0\vec{y}_0$
 $= R\sin\theta - L\cos\beta - 0$

$$\begin{cases} \vec{OB} = x\vec{x}_0 \\ \vec{v}_{B/0} = \dot{x}\vec{x}_0 \\ = (1 - R\dot{\theta}\sin\theta + L\dot{\beta}\cos\beta)\vec{x}_0 \end{cases} \quad \begin{aligned} \text{On a } x &= R\cos\theta + L\sin\beta \\ \dot{x} &= -R\dot{\theta}\sin\theta + L\dot{\beta}\cos\beta \end{aligned}$$

On a $\cos\beta = \frac{R}{L}\sin\theta$

$$\begin{aligned} \vec{v}_{B/0} &= \left(-R\dot{\theta}\sin\theta + L \cdot \frac{-R\dot{\theta}\cos\theta}{\sqrt{1 - \frac{R^2}{L^2}\sin^2\theta}} \cdot \frac{R\sin\theta}{L} \right) \vec{x}_0 \\ &= -R\dot{\theta}\sin\theta \left(1 - \frac{R}{L} \frac{\cos\theta}{\sqrt{1 - \frac{R^2}{L^2}\sin^2\theta}} \right) \vec{x}_0 \end{aligned}$$



$$1) \vec{OC}_1 = R\vec{y}_0 + (R + 2R\sin\theta)\vec{x}_0$$

$$\vec{V}_{C_1/0} = 2R\dot{\theta}\cos(\theta)\vec{x}_0$$

$$\vec{OC}_2 = R\vec{x}_0 + (R + 2R\cos\theta)\vec{y}_0$$

$$\vec{V}_{C_2/0} = -2R\dot{\theta}\sin(\theta)\vec{y}_0$$

$$\vec{V}_{C_2/S_0} = \vec{V}_{C_1/S_0} + \vec{C_2C_1} \wedge \vec{\Omega}_{S_0}$$

$$= 2R\dot{\theta}\cos(\theta)\vec{x}_0 - 2R\vec{x}_s \wedge \dot{\theta}\vec{z}_0$$

$$= 2R\dot{\theta}\cos(\theta)\vec{x}_0 - 2R\dot{\theta}\cos(\theta)\vec{x}_0 - 2R\dot{\theta}\sin\theta\vec{y}_0$$

$$2) \vec{V}_{J \in 1/0} = \vec{V}_{C_1 \in 1/0} + \vec{JC_1} \wedge \vec{\Omega}_{1/0}$$

Roulement sans glissement $\Rightarrow \vec{V}_{J \in 1/0} = 0$

$$\text{donc } \vec{V}_{C_1 \in 1/0} + \vec{JC_1} \wedge \vec{\Omega}_{1/0} = 0$$

$$2R\dot{\theta}\cos\theta\vec{x}_0 = -R\vec{y}_0 \wedge \omega_1\vec{z}_0$$

$$\downarrow \omega$$

$$= -R\omega_1\vec{x}_0$$

$$\omega_1 = -2\omega\cos\theta$$

$$\vec{V}_{PE211} = \vec{V}_{PE210} - \vec{V}_{PE110}$$

$$\begin{aligned}\vec{V}_{PE210} &= \vec{V}_{CE210} + \vec{PC}_2 \wedge \vec{\Omega}_{210} \\ &= -2R\omega \sin \theta \vec{y}_0 + R\vec{x}_3 \wedge \omega_2 \vec{z}_0 \\ &= -2R\omega \sin \theta \vec{y}_0 - \omega_2 R \vec{y}_5\end{aligned}$$

On projette $\vec{y}_5$:

$$\begin{cases} -\cos \theta \vec{x}_0 \\ -\sin \theta \vec{y}_0 \end{cases}$$

$$\begin{aligned}\vec{V}_{PE210} &= -2R\omega \sin \theta \vec{y}_0 + (\omega_2 R (\cos \theta \vec{x}_0 + \sin \theta \vec{y}_0)) \\ &= -2R\omega \sin \theta \vec{y}_0 + \omega_2 R \cos \theta \vec{x}_0 + \omega_2 R \sin \theta \vec{y}_0 \\ &= \begin{cases} \omega_2 R \sin \theta \vec{y}_0 \\ \omega_2 R \cos \theta \vec{x}_0 \\ \omega_2 R \sin \theta - 2R\omega \sin \theta \vec{y}_0 \end{cases}\end{aligned}$$

$$\begin{aligned}\vec{V}_{PE110} &= \vec{V}_{CE110} + \vec{PC}_1 \wedge \vec{\Omega}_{110} \\ &= 2R\dot{\theta} \cos \theta \vec{x}_0 - R\vec{x}_3 \wedge \omega_1 \vec{z}_0 \\ &= 2R\omega \cos \theta \vec{x}_0 - R(-\sin \theta \vec{x}_0 + \cos \theta \vec{y}_0) \wedge \omega_1 \vec{z}_0 \\ &= 2R\omega \cos \theta \vec{x}_0 - R\omega_1 (\cos \theta \vec{x}_0 + \sin \theta \vec{y}_0) \\ &= \begin{cases} 2R\omega \cos \theta - R\omega_1 \cos \theta \vec{x}_0 \\ -R\omega_1 \sin \theta \vec{y}_0 \end{cases}\end{aligned}$$

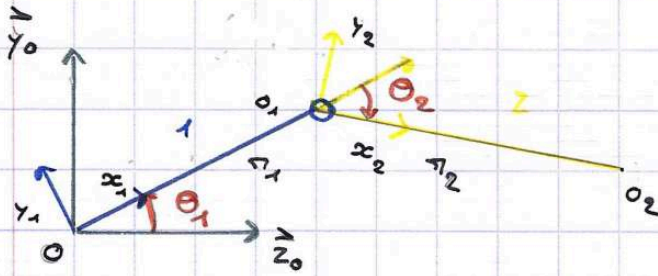
On a $\vec{V}_{PE211} = \vec{V}_{PE210} - \vec{V}_{PE110}$

" 0

$$\begin{cases} \omega_2 R \cos \theta \\ \omega_2 R \sin \theta - 2R\omega \sin \theta \end{cases} = \begin{cases} 2R\omega \cos \theta - R\omega_1 \cos \theta \\ -R\omega_1 \sin \theta \end{cases}$$

$$\begin{cases} \omega_2 = 2\omega - \omega_1 & \Leftrightarrow \omega_2 = 2\omega + 2\cos \theta \omega \\ \omega_1 = 2\omega - \omega_2 & = 2\omega (1 + \cos \theta) \end{cases}$$

La gauche :



$$\{V_{20}\} = \{V_{21}\} + \{V_{10}\}$$

$$\{V_{10}\} = \begin{cases} \vec{\Omega}_{10} = \dot{\theta}_1 \vec{z} \\ \vec{V}_{O_1 \in 1/0} = \vec{0} \end{cases} \rightarrow \begin{cases} \vec{\Omega}_{10} \\ \vec{V}_{O_1 \in 1/0} = \vec{V}_{O_1 \in 0/0} + \vec{O_1 O} \wedge \vec{\Omega}_{10} \end{cases}$$

$$\{V_{21}\} = \begin{cases} \vec{\Omega}_{21} = \dot{\theta}_2 \vec{z} \\ \vec{V}_{O_2 \in 2/1} = \vec{0} \end{cases}$$

$$\text{donc } \{V_{20}\} = \{V_{21}\} + \{V_{10}\} = \begin{cases} \vec{\Omega}_{20} = (\dot{\theta}_2 + \dot{\theta}_1) \vec{z} \\ \vec{V}_{O_2 \in 2/0} = \vec{O_1 O} \wedge \vec{\Omega}_{10} \\ = -R_1 \vec{x}_1 \wedge \dot{\theta}_1 \vec{z} \\ = R_1 \dot{\theta}_1 \vec{y}_1 \end{cases}$$

$$\rightarrow \begin{cases} \vec{\Omega}_{20} \neq \\ \vec{V}_{O_2 \in 2/0} = \vec{V}_{O_2 \in 2/1} + \vec{O_2 O_1} \wedge \vec{\Omega}_{20} \\ = R_1 \dot{\theta}_1 \vec{y}_1 + (-R_2 \vec{x}_2) \wedge (\dot{\theta}_2 + \dot{\theta}_1) \vec{z} \\ = R_1 \dot{\theta}_1 \vec{y}_1 + R_2 (\dot{\theta}_2 + \dot{\theta}_1) \vec{y}_2 \end{cases}$$

Par dérivation :

$$\begin{aligned} \vec{a}_{O_2 \in 2/0} &= \frac{d_0}{dt} \vec{V}_{O_2 \in 2/0} = \frac{d_0}{dt} (R_1 \dot{\theta}_1 \vec{y}_1 + R_2 (\dot{\theta}_2 + \dot{\theta}_1) \vec{y}_2) \\ &= R_1 (\ddot{\theta}_1 \vec{y}_1 + \dot{\theta}_1 \vec{\Omega}_{10} \wedge \vec{y}_1) + R_2 ((\ddot{\theta}_2 + \ddot{\theta}_1) \vec{y}_2 + (\dot{\theta}_1 + \dot{\theta}_2) (\vec{\Omega}_{20} \wedge \vec{y}_2)) \end{aligned}$$

$$= R_1 (\ddot{\theta}_1 \vec{y}_1 - \dot{\theta}_1^2 \vec{x}_1) + R_2 ((\ddot{\theta}_1 + \ddot{\theta}_2) \vec{y}_2 - (\dot{\theta}_2 + \dot{\theta}_1)^2 \vec{x}_2)$$

$$\vec{\gamma}_{Abs O_2} = \vec{\gamma}_{Ent O_2} + \vec{\gamma}_{Rel O_2} + \vec{\gamma}_{con O_2}$$

$$\vec{\gamma}_{Ent O_2} = \vec{\gamma}_{O_2 \in \Sigma 1-1} = \frac{d_1}{dt} (R_2 \dot{\theta}_2 \vec{y}_2)$$

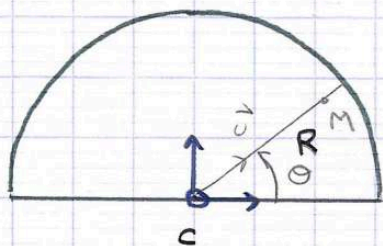
$$= R_2 (\ddot{\theta}_2 \vec{y}_2 + \dot{\theta}_2 \vec{\Omega}_{\Sigma 1-1} \wedge \vec{y}_2)$$

$$= R_2 (-\dot{\theta}_2^2 \vec{x}_2 + \ddot{\theta}_2 \vec{y}_2)$$

$$\vec{\gamma}_{con} = 2 \vec{\Omega}_{Ent} \wedge \vec{V}_{O_2 \in \Sigma 1-1}$$

$$\begin{aligned} \text{ici: } \vec{\gamma}_{con O_2} &= 2 \vec{\Omega}_{10} \wedge \vec{V}_{O_2 \in \Sigma 1-1} \\ &= 2 \dot{\theta}_1 \vec{z} \wedge R_2 \dot{\theta}_2 \vec{y}_2 \\ &= -2 R_2 \dot{\theta}_1 \dot{\theta}_2 \vec{x}_2 \end{aligned}$$

Recherche de centre de gravité



Le demi disque est homogène.

Gm est dans le plan de la feuille donc $z=0$

Symétrie par rapport à $Cy \Rightarrow x=0$

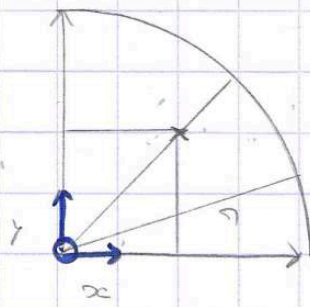
Donc G sur l'axe Cy

Théorème du barycentre: $m \vec{OG} = \int_{\Sigma} \vec{OM} \cdot dm$

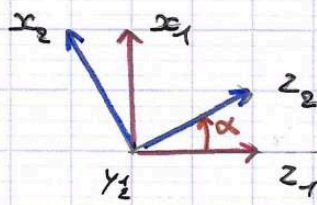
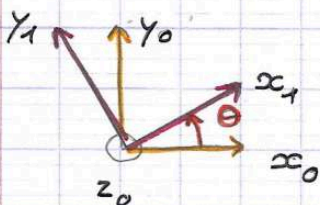
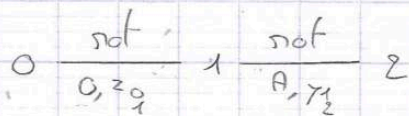
$$m \cdot \vec{CG} \cdot \vec{y} = \int \sigma r^2 dr d\theta (\vec{u} \cdot \vec{y})$$

$$m \cdot \vec{y}_G = \left(\int \sigma r^2 dr d\theta \right) \cdot \vec{y}$$

$$\Rightarrow \vec{y}_G = \frac{1}{m} \sigma \int_0^R r^2 dr \int_0^\pi \sin \theta d\theta = \frac{4R}{3\pi}$$



Exercice : centrifugeuse

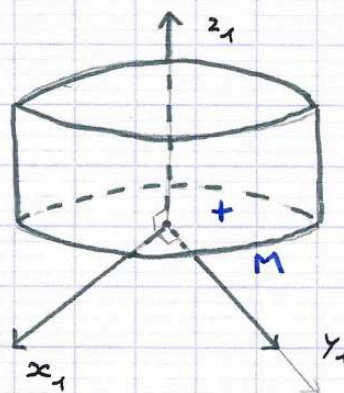


On cherche :

$$\{C_{10}\} = \begin{cases} \vec{P}_{10} = m_1 \vec{V}_{G_1 \in 1/0} \\ \vec{q}_{10} = \vec{H}_{O, b_1} \cdot \vec{\Omega}_{10} \end{cases}$$

$$\{V_{10}\} = \begin{cases} \vec{\Omega}_{10} = \dot{\Theta} \vec{z}_0 \\ \vec{V}_{O10} = \vec{0} = \vec{V}_{G10} \end{cases}$$

$$\vec{H}_{O, b_1} = \begin{bmatrix} A & -F & -E \\ & B & -D \\ & & C \end{bmatrix}_{O, b_1}$$



On passe en coordonnées cylindriques :

$$x = r \cos \Theta$$

$$y = r \sin \Theta$$

$$z = z$$

$$dm = \rho r d\Theta dr dz$$

$$A_0 = \int_1 (y^2 + z^2) dm$$

$$B_0 = \int_1 (z^2 + x^2) dm$$

$$C_0 = \int_1 (x^2 + y^2) dm$$

$$\begin{aligned} A_0 + B_0 &= \int_1 (x^2 + y^2 + 2z^2) dm \\ &= \int_1 (x^2 + y^2) dm + 2 \int_1 z^2 dm \\ &= C_0 + 2 \int_1 z^2 dm \end{aligned}$$

$$\begin{aligned} C_0 &= \int_1 r^2 \rho d\Theta dr dz = \rho \int_0^{2\pi} d\Theta \int_0^R r^3 dr \int_{-h/2}^{h/2} dz \\ &= K m_1 r^2 = \frac{m_1 R^2}{2} \end{aligned}$$

$$\int_1 z^2 dm = \int_1 z^2 \rho r \sin \theta dr d\theta dz = \rho \int_0^{2\pi} d\theta \int_{-h/2}^{h/2} z^2 dz \int_0^R r dr.$$

$$A_0 = B_0$$

$$2B_0 = 2A_0 = A_0 + B_0 = C_0 + 2 \int_1 z^2 dm$$

$$D = \int_1 yz dm = 0 \quad \text{car les fonctions sont impaires et le domaine est symétrique.}$$

idem E et F.

$$\text{Donc } \mathbb{I}_{O, b_1}^1 = \begin{bmatrix} A_0 & 0 & 0 \\ 0 & A_0 & 0 \\ 0 & 0 & C_0 \end{bmatrix}$$

$$\{C_{10}\} = \begin{cases} \vec{p}_{10} = \vec{0} \\ \vec{p}_{\alpha 10} = \mathbb{I}_{O, b_1}^1 \cdot \vec{\Omega}_{10} \end{cases} \quad \text{avec } \vec{\Omega}_{10} = \begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix}$$

$$= \begin{cases} \vec{p}_{10} = \vec{0} \\ \vec{p}_{\alpha 10} = 0 \vec{x}_1 + 0 \vec{y}_1 + C_0 \dot{\theta} \vec{z}_1 \end{cases}$$

$$\{C_{20}\} = \begin{cases} \vec{p}_{20} = m_2 \vec{V}_{G_2 20} \\ \vec{p}_{\alpha 20} = \mathbb{I}_{G_2, b_2}^2 \cdot \vec{\Omega}_{20} \end{cases}$$

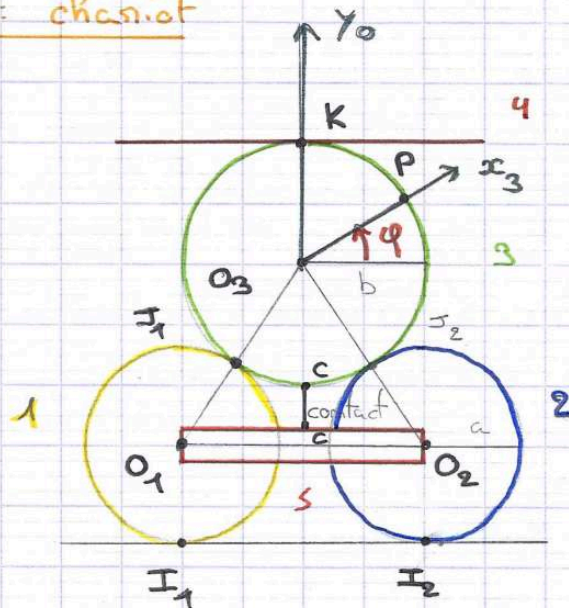
$$\begin{aligned} \{V_{20}\} = \begin{cases} \vec{\Omega}_{20} = \dot{\alpha} \vec{y}_2 + \dot{\theta} \vec{z}_1 = \dot{\alpha} \vec{y}_2 + \dot{\theta} (\cos \alpha \vec{z}_2 - \sin \alpha \vec{x}_2) \\ \quad = -\dot{\theta} \sin \alpha \vec{x}_2 + \dot{\alpha} \vec{y}_2 + \dot{\theta} \cos \alpha \vec{z}_2 \\ \vec{V}_{G_2 20} = \vec{V}_{A 20} + \vec{G_2 A} \wedge \vec{\Omega}_{20} \\ \quad = \vec{V}_{\alpha 10} + \vec{AO} \wedge \vec{\Omega}_{10} \end{cases} \end{aligned}$$

Avec les plans de symétrie et l'impairité des fonctions on a $\mathbb{I}_{G_2, b_2}^2$ une matrice diagonale.

Par symétrie de révolution on $B_{G_2} = C_{G_2}$

$A = \int (y^2 + z^2) dm$ les axes orthogonaux sont y et z
donc si on passe en cylindrique on a

Exercice : chariot



$$\vec{C}_{O_2} = \frac{\sqrt{2}}{2} (a+b) \vec{x}_0$$

$$\begin{aligned} \vec{C}_{O_1} &= -\vec{C}_{O_2} \\ \vec{C}_{O_1 O_3} &= \frac{\sqrt{2}}{2} (a+b) (\vec{x}_0 + \vec{y}_0) \\ \vec{C}_{O_2 O_3} &= \frac{\sqrt{2}}{2} (a+b) (-\vec{x}_0 + \vec{y}_0) \\ \vec{C}_{O_3} &= \frac{\sqrt{2}}{2} (a+b) \vec{y}_0 \end{aligned}$$

$$\left\{ \begin{array}{l} \vec{V}_{I_1} = \dot{\varphi}_1 \vec{z}_0 \\ \vec{V}_{O_1 \in I_1} = V \vec{x}_0 \end{array} \right\} \rightarrow \left\{ \begin{array}{l} \vec{V}_{I_1} \\ \vec{V}_{I_1 \in I_1} = \vec{V}_{I_1 \in I_1} + \vec{I}_1 \dot{\varphi}_1 \end{array} \right.$$

$$\begin{aligned} \vec{V}_{C \in I_1} &= V \vec{x}_0 \\ \vec{V}_{O_1 \in I_1} &= \\ \vec{V}_{O_1 \in I_1} &= \text{axe not} \end{aligned}$$

$$\begin{aligned} \vec{V}_{I_1 \in I_1} &= \vec{V}_{I_1 \in I_1} + \vec{I}_1 \dot{\varphi}_1 \\ &= V \vec{x}_0 + a \dot{\varphi}_1 \vec{z}_0 \\ &= (V + a \dot{\varphi}_1) \vec{x}_0 = \vec{0} \end{aligned}$$

$$\begin{aligned} \text{donc } \vec{V}_{I_1 \in I_1} &= -a \dot{\varphi}_1 \quad \dot{\varphi}_1 = \dot{\varphi}_2 \\ \vec{V}_{I_2 \in I_1} &= -\dot{\varphi}_2 a \end{aligned}$$

$$\begin{aligned}\vec{V}_{Me315} &= \underbrace{\vec{V}_{O_3 \in 315}}_0 + \vec{MO}_3 \wedge \vec{\Omega}_{315} \\ &= -b \dot{x}_3 \wedge \dot{\varphi}_3 \vec{z}_0 = b \dot{\varphi}_3 \vec{y}_3\end{aligned}$$

$$M \rightarrow J_1$$

$$\vec{V}_{J_1 \in 315} = b \dot{\varphi}_3 \frac{\vec{O_3 O_2}}{\|\vec{O_3 O_2}\|}$$

$$\vec{V}_{J_1 \in 315} = -b \dot{\varphi}_3 \frac{\sqrt{2}}{2} (a+b)(-\vec{x}_0 + \vec{y}_0)$$

$$\begin{aligned}\|\vec{O_3 O_2}\| &= \left(\frac{1}{2} (a+b)^2 + \frac{1}{2} (a+b)^2 \right)^{1/2} \\ &= a+b\end{aligned}$$

$$\vec{V}_{J_1 \in 315} = -b \dot{\varphi}_3 \frac{\sqrt{2}}{2} (\vec{y}_0 - \vec{x}_0)$$

$$J_1 \rightarrow J_3$$

$$\vec{V}_{J_1 \in 311} = \vec{0}$$

$$\vec{V}_{J_1 \in 315} - \vec{V}_{J_1 \in 115} = \vec{0}$$

$$\text{donc } \vec{V}_{J_1 \in 315} = \vec{V}_{J_1 \in 115}$$

$$\vec{V}_{J_1 \in 115} = \vec{V}_{O_1 \in 115} + \vec{J_1 O_1} \wedge \vec{\Omega}_{115}$$

$$= a \frac{\vec{O_1 O_3}}{\|\vec{O_1 O_3}\|} \wedge \dot{\varphi}_1 \vec{z}_0$$

$$= -a \frac{\sqrt{2}}{2} (a+b)(\vec{x}_0 + \vec{y}_0) \times (a+b)^{-1}$$

$$= -a \frac{\sqrt{2}}{2} (\vec{x}_0 + \vec{y}_0)$$

Si on prend l'égalité on a:
 $b \frac{\sqrt{2}}{2} \dot{\varphi}_3 (\vec{x}_0 - \vec{y}_0) = -a \frac{\sqrt{2}}{2} \dot{\varphi}_1 (\vec{x}_0 - \vec{y}_0)$
 donc $b \dot{\varphi}_3 = -a \dot{\varphi}_1$

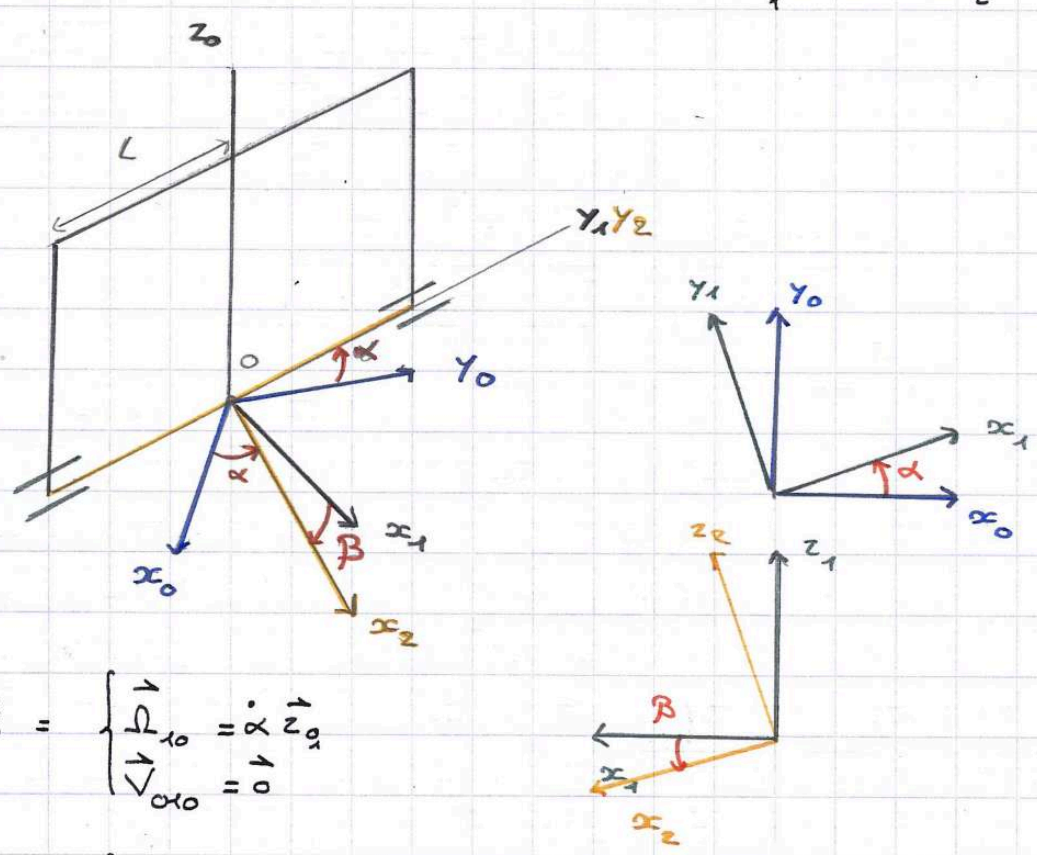
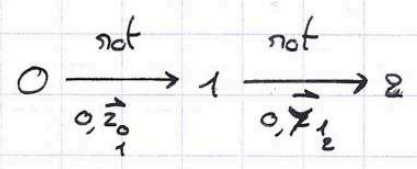
$K \rightarrow 3$ Roulement sans glissement $\vec{V}_{K \in 1/3} = \vec{0}$
 $\Rightarrow \vec{V}_{K \in 1/0} = \vec{V}_{K \in 3/0}$

$$\vec{V}_{O_3 \in 5/0} = \vec{V}_{\vec{x}_0}$$

$$\vec{V}_{O_3 \in 3/0} = \vec{V}_{O_3 \in 3/5} + \vec{V}_{O_3 \in 5/0} = \vec{V}_{\vec{x}_0}$$

$\vec{\gamma}_{S_1 \in 3/5}$ on calcule $\vec{\gamma}_{M \in 3/5}$

Cadre tournant



$$\{V_{1/0}\} = \begin{cases} \dot{p}_{1/0} = \dot{\alpha} \vec{z}_0 \\ \dot{\chi}_{1/0} = 0 \end{cases}$$

$$\{V_{2/1}\} = \begin{cases} \dot{p}_{2/1} = \dot{\beta} \vec{y}_1 \\ \dot{\chi}_{2/1} = 0 \end{cases}$$

$$\vec{z}_2 = \cos \beta \vec{z}_1 + \sin \beta \vec{y}_1$$

$$\vec{z}_1 = \cos \beta \vec{z}_2 + \sin \beta \vec{y}_2$$



1) a) $A \in 1 = A \in 2$

$$\begin{aligned} \vec{V}_{A \in 1/0} &= \vec{\Omega}_{1/0} \wedge \vec{OA} \\ &= \dot{\alpha} \vec{z}_1 \wedge -L \vec{y}_1 \\ &= L \dot{\alpha} \vec{x}_1 \end{aligned}$$

b) $\vec{\gamma}_{A \in 1/0} = \frac{d_0}{dt} (L \dot{\alpha} \vec{x}_1)$

$$\begin{aligned} &= L (\ddot{\alpha} \vec{x}_1 + \dot{\alpha} \frac{d_0 \vec{x}_1}{dt}) \\ &= L (\ddot{\alpha} \vec{x}_1 + \dot{\alpha} \vec{\Omega}_{1/0} \wedge \vec{x}_1) \\ &= L (\ddot{\alpha} \vec{x}_1 + \dot{\alpha}^2 \vec{y}_1) \end{aligned}$$

2) a) $\vec{V}_{P \in 2/1} = \vec{\Omega}_{2/1} \wedge \vec{OP}$

$$\begin{aligned} &= \dot{\beta} \vec{y}_2 \wedge \rho \vec{x}_2 \\ &= -\rho \dot{\beta} \vec{z}_2 \end{aligned}$$

b) $\vec{\gamma}_{P \in 2/1} = \frac{d_1 \vec{V}_{P \in 2/1}}{dt} = -\rho \frac{d_1}{dt} (\dot{\beta} \vec{z}_2)$

$$\begin{aligned} &= -\rho (\ddot{\beta} \vec{z}_2 + \dot{\beta} \vec{\Omega}_{2/1} \wedge \vec{z}_2) \\ &= -\rho (\ddot{\beta} \vec{z}_2 + \dot{\beta}^2 \vec{x}_2) \end{aligned}$$

3) a) $\vec{\gamma}_{abs P} = \vec{\gamma}_{ent P} + \vec{\gamma}_{rel P} + \vec{\gamma}_{cor P}$

$$\vec{\gamma}_{ent P} = \vec{\gamma}_{abs P} - \vec{\gamma}_{rel P} + \vec{\gamma}_{cor P}$$

$$\vec{\gamma}_{cor P} = 2 \vec{\Omega}_{1/0} \wedge \vec{V}_{P \in 2/1}$$

$$\begin{aligned} &= 2 \dot{\alpha} \vec{z}_1 \wedge (-\rho \dot{\beta} \vec{z}_2) \\ &= -2 \rho \dot{\alpha} \dot{\beta} (\vec{z}_1 \wedge \vec{z}_2) \\ &= -2 \rho \dot{\alpha} \dot{\beta} \sin \beta \vec{y}_2 \end{aligned}$$

$$\begin{aligned} \vec{V}_{P \in 1/0} &= \vec{V}_{O \in 1/0} + \vec{\Omega}_{1/0} \wedge \vec{OP} \\ &= \dot{\alpha} \vec{z}_1 \wedge \rho \vec{x}_2 \\ &= \dot{\alpha} \rho \cos \beta \vec{y}_2 \end{aligned}$$

$$\vec{\gamma}_{P \in 1/0} = \frac{d_0}{dt} (\rho \dot{\alpha} \cos \beta \vec{y}_2)$$

$$\begin{aligned} &= \rho ([\ddot{\alpha} \cos \beta + \dot{\alpha} - \dot{\beta} \sin \beta] \vec{y}_2 \\ &\quad + \dot{\alpha} \cos \beta (\dot{\beta} \vec{y}_2 + \dot{\alpha} \vec{z}_1) \wedge \vec{y}_2) \\ &= \rho ([\ddot{\alpha} \cos \beta - \dot{\alpha} \dot{\beta} \sin \beta] \vec{y}_2 \\ &\quad - \dot{\alpha}^2 \cos \beta \vec{x}_1) + 2 (\dot{\beta}^2 \vec{x}_2 + \ddot{\beta} \vec{z}_2) \\ &\quad + 2 \rho \dot{\alpha} \dot{\beta} \sin \beta \vec{y}_2 \end{aligned}$$

$$\vec{x}_1 = \cos \beta \vec{x}_2 + \sin \beta \vec{z}_2$$

Home (examen) (Annale 2014-2015)

9

$$2) 3) 1) \quad \left\{ \begin{array}{l} \vec{V}_{10} = \dot{\vec{r}}_{10} = \dot{\alpha} \vec{z}_0 \\ \vec{V}_{C10} = \vec{0} \end{array} \right. \rightarrow \left\{ \begin{array}{l} \dot{\alpha} \vec{z}_0 \\ \vec{V}_{I \in 110} = \vec{r}_{10} \wedge \vec{\Omega} \\ = \eta_1 \dot{\alpha} \vec{u}_1 \end{array} \right.$$

$$4) \vec{V}_{P \in 110} = \vec{r}_{10} \wedge \vec{\Omega} = \dot{\alpha} \vec{z}_0 \wedge \eta_1 \vec{x}_1 = \eta_1 \dot{\alpha} \vec{y}_1$$

$$5) \vec{V}_{P \in 110} = \frac{d}{dt} \vec{OP} = \eta_1 \frac{d \vec{x}_1}{dt} = \eta_1 (\vec{r}_{10} \wedge \vec{x}_1) = \eta_1 \dot{\alpha} \vec{y}_1$$

$$8) \vec{\gamma}_{P \in 110} = \frac{d}{dt} (\eta_1 \dot{\alpha} \vec{y}_1) = \eta_1 (\ddot{\alpha} \vec{y}_1 + \dot{\alpha} \frac{d \vec{y}_1}{dt}) = \eta_1 (-\dot{\alpha}^2 \vec{x}_1 + \ddot{\alpha} \vec{y}_1)$$

$$9) \vec{\gamma}_{P \in 110} \cdot \vec{x}_1 = -\eta_1 \dot{\alpha}^2 \quad \cdot \text{pas d'accélération tangentielle}$$

$\cdot -\eta_1 \dot{\alpha}^2 < 0$ donc l'accélération est radiale et centrale.

$$10) 11) 12) \quad \left\{ \begin{array}{l} \vec{r}_{20} = \dot{\theta} \vec{z}_0 \\ \vec{V}_{A \in 210} = \vec{0} \end{array} \right.$$

$$13) \vec{V}_{Q \in 210} = \vec{r}_{20} \wedge \vec{\Omega} = \dot{\theta} \vec{z}_0 \wedge \eta_2 \vec{x}_2 = \eta_2 \dot{\theta} \vec{y}_2$$

$$14) \vec{V}_{I \in 210} = -\eta_2 \dot{\theta} \vec{u}$$

$$15) \quad \begin{array}{c} \nearrow 2 \\ \searrow 1 \end{array} \quad \vec{V}_{I \in 211} = \vec{V}_{I \in 210} - \vec{V}_{I \in 110}$$

$$16) \quad \vec{0} = \vec{V}_{I \in 210} - \vec{V}_{I \in 110}$$

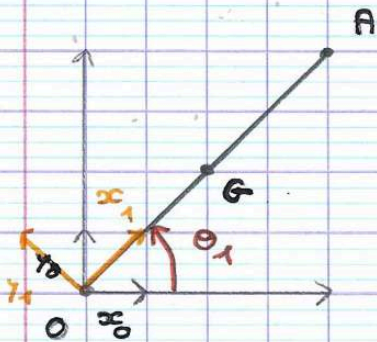
$$17) \quad \dot{\theta} \eta_2 = -\dot{\alpha} \eta_1$$

$$18) \text{ De manière analogue } \dot{\beta} \eta_2 = \dot{\alpha} \eta_1$$

$$19) C_{A20} = \int (\alpha^2 + \gamma^2) dm = \int \eta^2 dm = \int \eta^2 \rho dV = \rho \int_0^{\eta_2} \eta^3 d\eta \int_0^{2\pi} d\theta \int_{-L/2}^{L/2} dz = \frac{\eta_1^2}{2} \rho L \pi$$

$$\begin{aligned}
 C_{\text{c20}} &= \int_1 (x^2 + y^2) dm \\
 &= \int_1 r_1^2 \lambda r_1 d\theta = r_1^3 \lambda 2\pi = m_1 r_1^2
 \end{aligned}$$

Utilisation du théorème de Huygens : la gauche



$$\vec{I}_{G \rightarrow 10} = \vec{I}_{G \rightarrow 1} \cdot \vec{I}_{1 \rightarrow 10}$$

$$\vec{I}_{O \rightarrow 10} = \vec{I}_{O \rightarrow 1} \cdot \vec{I}_{1 \rightarrow 10}$$

$$\vec{I}_{O \rightarrow 10} = \vec{I}_{G \rightarrow 10} + \vec{OG} \wedge \vec{P}_{10}$$

$$\begin{aligned} C_{G \rightarrow 1} &= \int_1 (y^2 + x^2) dm \\ &= \lambda \int_0^{L/2} x^2 dx = \frac{\lambda}{12} L^3 = \frac{1}{12} m_1 L^2 \end{aligned}$$

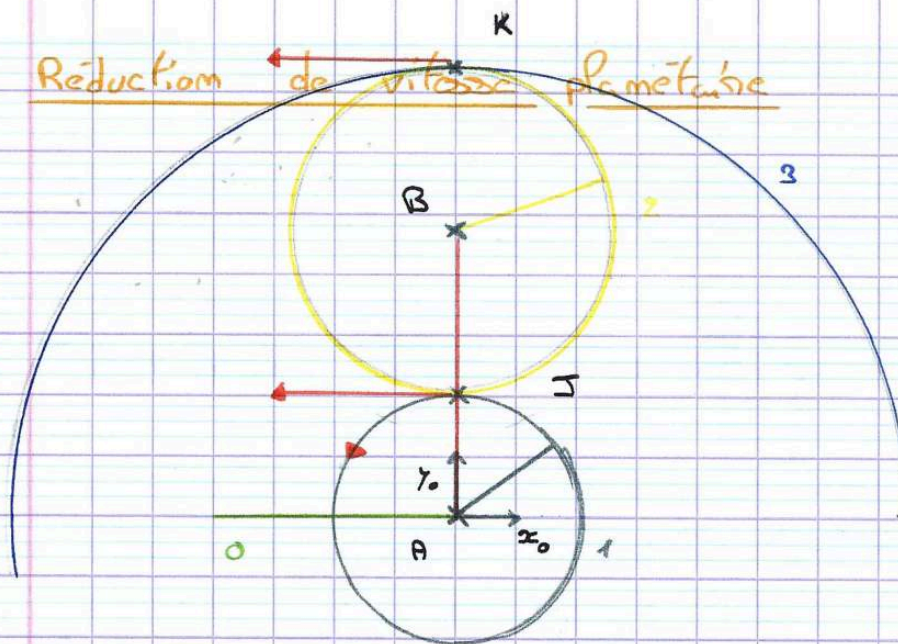
$$\text{donc } \vec{I}_{G \rightarrow 10} = C_{G \rightarrow 1} \dot{\theta}_1 \vec{z}_0$$

$$\begin{aligned} \vec{I}_{O \rightarrow 10} &= \vec{I}_{G \rightarrow 10} + \vec{OG} \wedge \vec{P}_{10} \\ &= \vec{I}_{G \rightarrow 10} + \frac{L}{2} \vec{x}_1 \wedge m_1 \frac{L}{2} \dot{\theta}_1 \vec{y}_1 \\ &= \dot{\theta}_1 \left(C_G + m_1 \frac{L^2}{4} \right) \vec{z}_0 \end{aligned}$$

$$\vec{I}_{O \rightarrow 10} = \begin{bmatrix} 0 \\ 0 \\ C_O \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} = C_O \dot{\theta}_1 \vec{z}_0$$

$$C_O = \int_1 y^2 + x^2 dm = \int_0^L x^2 \lambda dx = \lambda \frac{L^3}{3} = \frac{m L^2}{3}$$

$$\text{Par Huygens : } C_O = C_G + m \frac{L^2}{4} = \frac{m L^2}{12} + \frac{3 m L^2}{12} = \frac{m L^2}{3} \checkmark$$



$$\begin{aligned} \vec{v}_{B/10} &= -\eta_1 \dot{\theta}_{10} \vec{x}_0 \\ \vec{v}_{B/10} &= (\eta_1 + \eta_2) \dot{\theta}_{10} \vec{x}_0 + \eta_2 \dot{\theta}_{20} \vec{x}_0 \end{aligned}$$

$$\begin{aligned} \vec{v}_{B/1} &= \vec{v}_{B/10} - \vec{v}_{A/10} \\ &= \vec{v}_{B/10} - \vec{v}_{A/10} \end{aligned}$$

$$0 = -(\eta_1 + \eta_2) \omega_{10} + \eta_2 \omega_{20} + \eta_1 \omega_{10}$$

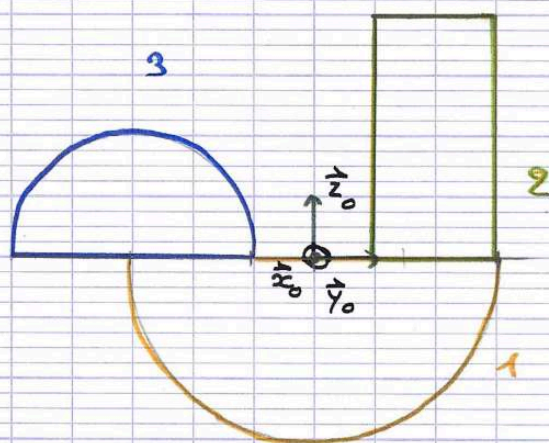
$$\begin{aligned} \vec{v}_{K/2} &= \vec{v}_{K/10} - \vec{v}_{A/10} \\ &= -\eta_3 \omega_3 \vec{x}_0 + (\eta_1 + \eta_2) \omega_{10} \vec{x}_0 + \eta_2 \omega_{20} \vec{x}_0 \\ \left\{ \begin{aligned} 0 &= -\eta_3 \omega_3 + (\eta_1 + \eta_2) \omega_{10} + \eta_2 \omega_{20} \\ 0 &= \eta_1 \omega_{20} - (\eta_1 + \eta_2) \omega_{10} + \eta_2 \omega_{20} \end{aligned} \right. \end{aligned}$$

$$\Rightarrow \frac{\omega_3}{\omega_{10}} = -\frac{\eta_1}{\eta_3} \quad \text{si } \omega_{20} = \omega_{10} \Rightarrow \omega_3 = \omega_{10}$$

$$\text{si } \omega_{20} \neq \omega_{10} \Rightarrow \frac{\omega_{10} - \omega_{10}}{\omega_{20} - \omega_{10}} = -\frac{\eta_3}{\eta_1} \quad (\text{relation de Willis})$$

$$\begin{aligned}\vec{\gamma}_{Jez10} &= \frac{d_0}{dt} \vec{\gamma}_{Jez10} = \frac{d_0}{dt} \left([-(\eta_1 + \eta_2) \omega_{10} + \eta_2 \omega_{20}] \vec{x}_0 \right) \\ &= \vec{x}_0 \left[-(\eta_1 + \eta_2) \dot{\omega}_{10} + \eta_2 \dot{\omega}_{20} \right] \\ &= \vec{x}_0 \left[-(\eta_1 + \eta_2) \dot{\omega}_{10} + \eta_2 \dot{\omega}_{20} \right] + [-(\eta_1 + \eta_2) \omega_{10} + \eta_2 \omega_{20}] \vec{\Omega}_{20} \wedge \vec{x}_0\end{aligned}$$

Girovettie parte

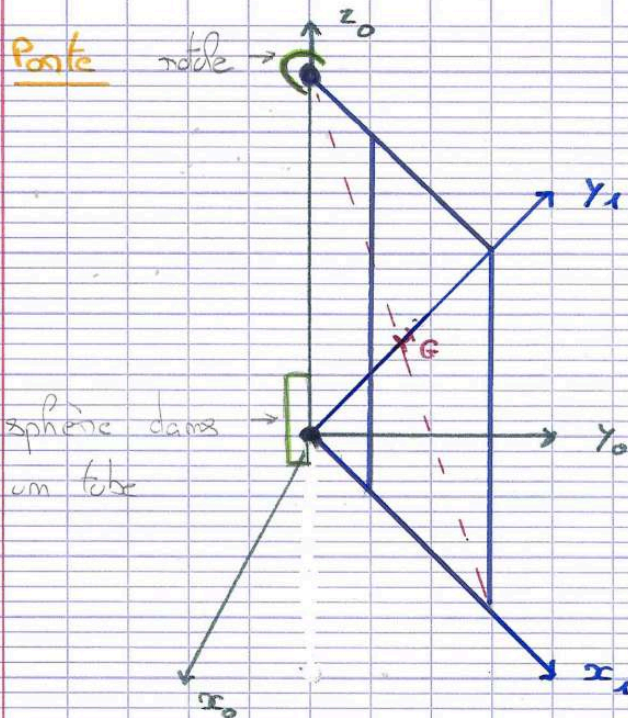


$$\begin{aligned}I_{O, \vec{z}_0}^{1+2} &= I_{O, \vec{z}_0}^1 + I_{O, \vec{z}_0}^2 \\ &\downarrow \quad \quad \downarrow \\ &1/2 \text{ sphère} \quad \text{rectangle} \\ &\text{sphériques} \quad \text{cartésiennes}\end{aligned}$$

$$I_{O, \vec{z}_0}^{1+2} = I_{O, \vec{z}_0}^{1+2} - m_{1+2} c^2$$

$$\begin{aligned}I_{O, \vec{z}_0}^2 &= \int_V (x^2 + y^2) dm \\ &= \sigma \int_{R/2}^R y^2 dy \int_0^R dz\end{aligned}$$

$$\begin{aligned}I_{O, \vec{z}_0}^1 &= \int_V (x^2 + y^2) \rho dV = \rho \int_V r^2 \sin^2 \theta d\theta d\phi dr \\ &= \rho \int_0^R r^4 dr \int_0^{2\pi} d\phi \int_{\pi/2}^{\pi} \sin^3 \theta d\theta \\ &= \rho \int_0^R r^4 dr \int_0^{2\pi} d\phi \int_{\pi/2}^{\pi} \sin \theta d\theta - \cos^2 \theta \sin \theta d\theta \\ &= \rho \int_0^R r^4 dr \int_0^{2\pi} d\phi \int_{\pi/2}^{\pi} \sin \theta d\theta - \int_{\pi/2}^{\pi} \cos^2 \theta \sin \theta d\theta\end{aligned}$$



$$\# [A_{\alpha}]_A = [\vec{R}_{\alpha}] = x_{\alpha} \vec{x}_1 + y_{\alpha} \vec{y}_1 + z_{\alpha} \vec{z}_1 ; \vec{x}_{A\alpha} = \vec{0}]$$

$$= \begin{bmatrix} x_{\alpha} & 0 \\ y_{\alpha} & 0 \\ z_{\alpha} & 0 \end{bmatrix}$$

$$[A'_{\alpha}]_B = [\vec{R}'_{\alpha}] = x'_{\alpha} \vec{x}_1 + y'_{\alpha} \vec{y}_1 + 0 \vec{z}_1 ; \vec{M}'_{B\alpha} = \vec{0}]$$

$$= \begin{bmatrix} x'_{\alpha} & 0 \\ y'_{\alpha} & 0 \\ z'_{\alpha} & 0 \end{bmatrix}$$

$$[G_{oom}] = [\vec{0}, -R\dot{\theta}\vec{z}_0]$$

$$[P_r] = [-mg\vec{z}_0, \vec{0}]$$

$$[A_{\alpha}][V_{10}] = \begin{bmatrix} x_{\alpha} \vec{x}_1 + y_{\alpha} \vec{y}_1 + z_{\alpha} \vec{z}_1 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} \vec{p}_{10} \\ 0 \\ 0 \end{bmatrix}$$

$$= 0$$

$$\begin{aligned}\vec{q}_{oe210} &= \vec{q}_{pe210} + \vec{q}_{pe21} \vec{OP} \wedge \vec{p}_{20} = (z_p \vec{z} + x_p \vec{x}_1) \wedge \mu x_p \dot{\theta} \vec{y}_1 \\ &= \mu x_p \dot{\theta} (-z_p \vec{x}_1 + x_p \vec{z})\end{aligned}$$

$$\begin{aligned}\vec{q}_{oe210} &= \mu x_p \ddot{\theta} (-z_p \vec{x}_1 + x_p \vec{z}) + \mu x_p \dot{\theta}^2 \vec{y}_1 \\ \vec{q}_{oe210} &= \begin{cases} -E\ddot{\theta} - \mu x_p z_p \ddot{\theta} & \vec{x}_1 \\ -E\dot{\theta}^2 - \mu x_p z_p \dot{\theta}^2 & \vec{y}_1 \\ C\ddot{\theta} + \mu x_p^2 \ddot{\theta} & \vec{z} \end{cases}\end{aligned}$$

$$\vec{S}_{oe210} = -L(\ddot{\theta} \vec{x}_1 + \ddot{\theta} \vec{y}_1) + C\ddot{\theta} \vec{z}$$

$$\text{donc } \vec{S}_{oe210} \cdot \vec{x}_1 = -\ddot{\theta}(L + \mu x_p z_p) = 0 \Rightarrow z_p = \frac{-L}{\mu x_p}$$

équilibre dynamique

Vitesse

$$\{V_{30}\} = \begin{cases} \vec{L}_{30} \\ \vec{v}_{G \in 310} = \dot{x} \vec{x}_1 \end{cases}$$

PFD à 310

$$\{D_{30}\} = \{A_{310}\}$$

$$\{C_{30}\} = \begin{cases} \vec{p}_{30} = m \dot{x} \vec{x}_1 \\ \vec{q}_{G \in 310} = \vec{0} \end{cases}$$

$$\underbrace{\vec{O}_2 \vec{O}_1 \wedge (-m_2 g \vec{y})}_{=0} + \underbrace{\vec{O}_2 \vec{O}_1 \wedge \vec{F}_{32}}_{=0} + \underbrace{\vec{O}_2 \vec{I}_2 \wedge \vec{F}_{O_2}}_{=0} = \vec{0}$$

$$\{D_{30}\} = \begin{cases} \vec{d}_{30} = m \dot{x} \vec{x}_1 \\ \vec{q}_{G \in 310} = \vec{0} \end{cases}$$

$$\text{Donc } \vec{O}_2 \vec{I}_2 \wedge \vec{F}_{O_2} = \vec{0}$$

1) Isolation + Bilan des actions

$$\text{en } O \text{ poids } \rightarrow \vec{p} = -mg \vec{y} \rightarrow \vec{m}_{G \in 10} = \vec{OG} \wedge \vec{p} = \vec{0}$$

$$\text{en } I_2, A_{O_2} \rightarrow \vec{F}_{O_2} = Y_2 \vec{y} \rightarrow \vec{m}_{O_2 \in 02} = \vec{O_2 I_2} \wedge Y_2 \vec{y} = -P_2 Y_2 \vec{z}$$

$$\begin{aligned}\text{en } I_1, A_{O_1} \rightarrow \vec{F}_{O_1} &= X_1 \vec{x} + Y_1 \vec{y} \rightarrow \vec{m}_{O_1 \in 01} = \vec{G I_1} \wedge (X_1 \vec{x} + Y_1 \vec{y}) \\ &= (-h \vec{y} + P_1 \vec{x}) \wedge (X_1 \vec{x} + Y_1 \vec{y}) \\ &= (P_1 Y_1 + h X_1) \vec{z}\end{aligned}$$

$$\begin{cases} m \ddot{\vec{x}} = \begin{cases} -mg \vec{y} + Y_2 \vec{y} + Y_1 \vec{y} + X_1 \vec{x} \\ \vec{0} = \begin{cases} -P_2 Y_2 \vec{z} + (P_1 Y_1 + h X_1) \vec{z} \end{cases} \end{cases} \end{cases}$$

$$m \ddot{x} = X_1 \quad (\vec{x})$$

$$0 = -mg + Y_1 + Y_2 \quad (\vec{y})$$

$$0 = -P_2 Y_2 + P_1 Y_1 + h X_1 \quad (\vec{z})$$

$$\frac{X_1}{Y_1} = v$$

$$\ddot{x} = a$$

$$0 = -mg P_2 + Y_1 P_2 + Y_2 P_2$$

$$0 = h X_1 + Y_1 P_1 - Y_2 P_2$$

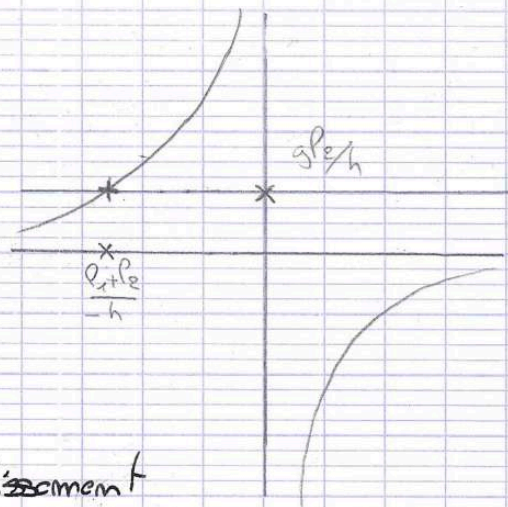
$$0 = -\frac{mg P_2}{X_1} + \frac{Y_2}{X_1} (P_2 + P_1) + h = 0$$

$$\Rightarrow -\frac{mg P_2}{m a} + \frac{1}{v} (P_2 + P_1) + h = 0$$

$$\Rightarrow \frac{1}{v} (P_2 + P_1) = \frac{g P_2}{a} - h$$

$$\Rightarrow \frac{1}{v} = \frac{g P_2}{a (P_2 + P_1)} - \frac{h}{P_2 + P_1}$$

$$\Rightarrow v = \frac{P_1 + P_2}{\frac{g P_2}{a} - h}$$

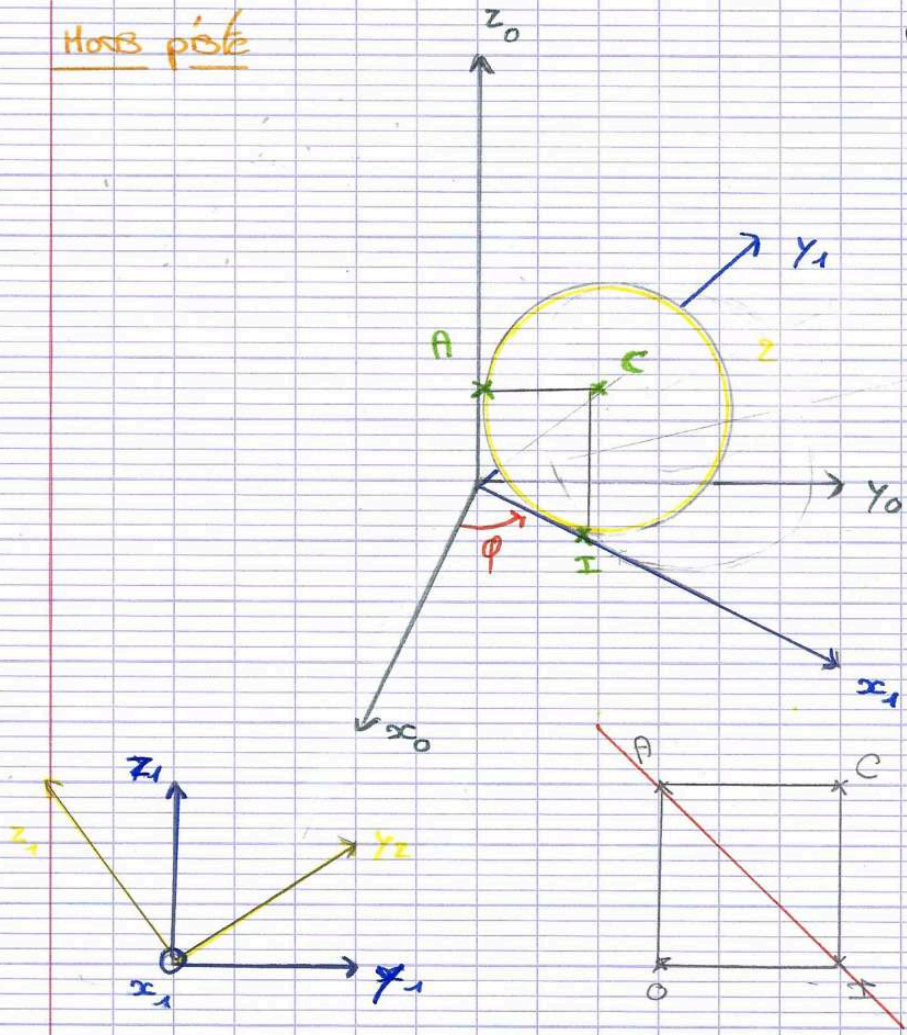


Poi de Coulomb: P_{limite de glissement}

$$g = \tan \varphi = \frac{X_1}{Y_1} = \frac{P_1 + P_2}{\frac{g P_2}{a} - h}$$

Rolling wheel

Rolling without slipping at A and I.



instantaneous axis of rotation of $\vec{\Omega}_{20}$

$$\begin{aligned} \vec{V}_{A \in 2/0} &= 0 \\ \vec{V}_{I \in 2/0} &= 0 \\ \vec{V}_{A \in 2/0} &= \vec{V}_{I \in 2/0} + \vec{AI} \wedge \vec{\Omega}_{20} \\ \Rightarrow \vec{\Omega}_{20} &= \lambda \vec{AI} \end{aligned}$$

$$\begin{aligned} \dot{\theta} &= \lambda R \\ \dot{\phi} &= -\lambda R \\ \dot{\theta} &= -\dot{\phi} \end{aligned}$$

$$\{V_{20}\}_G = \begin{cases} \vec{\Omega}_{20} = \dot{\theta} \vec{x}_1 + \dot{\phi} \vec{z}_1 = \lambda (-R \vec{z}_1 + R \vec{x}_1) = \dot{\phi} (-\vec{x}_1 + \vec{z}_1) \\ \vec{V}_{G \in 2/0} = \vec{\Omega}_{20} \wedge \vec{AG} = R \dot{\phi} \vec{y}_1 \end{cases}$$

$$\{C_{20}\}_G = \begin{cases} \vec{p}_{20} = m R \dot{\phi} \vec{y}_1 \\ \vec{G}_{2/0} = \vec{I}_G \cdot \vec{\Omega}_{20} \\ = \vec{I}_G \cdot \vec{\Omega}_{20} \end{cases} = \begin{bmatrix} A & & \\ & A & \\ & & A \end{bmatrix} \begin{bmatrix} -\dot{\phi} \\ 0 \\ \dot{\phi} \end{bmatrix}$$

$$\vec{\sigma}_{GE210} = A\dot{\varphi}(-\vec{x}_1 + \vec{z}_1)$$

$$A = \frac{2}{5} m r^2$$

$$\{D_{20}\}_C = \begin{cases} \vec{d}_{20} = m \vec{a}_{GE210} = \frac{d}{dt} (R\dot{\varphi}\vec{y}_1) = R(\ddot{\varphi}\vec{y}_1 + \dot{\varphi}\vec{\Omega}_{10} \wedge \vec{y}_1) \\ \vec{\sigma}_{GE210} = \frac{d}{dt} (\vec{\sigma}_{GE210}) = A\ddot{\varphi}(-\vec{x}_1 + \vec{z}_1) - A\dot{\varphi}\vec{y}_1 \end{cases}$$

Principe fondamental de la dynamique

$$\{D_{20}\} = \{A\vec{z}_{12}\}$$

Théorème de l'énergie cinétique

$$\frac{d\bar{E}_{GE210}}{dt} = P_{ext} \bar{z}_{12}$$



0 - 1 - 2

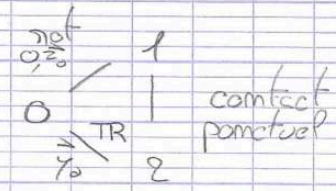
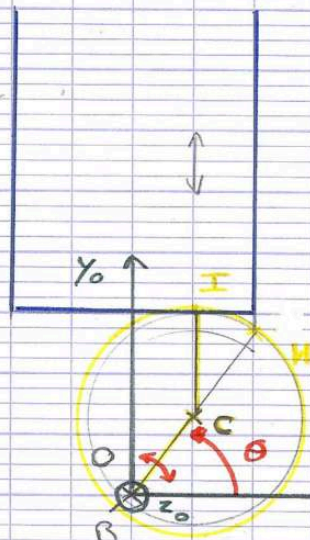
$$\{V_{20}\} = \{V_{21}\} + \{V_{10}\}$$

Pas la composition des accélérations

$$\{C_{20}\} = \begin{cases} mV_{GE210} + mV_{GE10} \\ I\vec{\Omega}_{21} + I\vec{\Omega}_{10} \end{cases}$$

$$\{D_{20}\} : \text{Con'd's}$$

Rotation géométrique - personnelle

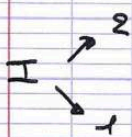


$$\{V_{10}\} = \begin{cases} \vec{h}_{10} = \dot{\theta} \vec{z} \\ \vec{v}_{O \in 10} = \vec{0} \end{cases}$$

$$\vec{v}_{C \in 10} = \vec{v}_{O \in 10} + \vec{h}_{10} \wedge \vec{OC} = e \dot{\theta} \vec{y}_1$$

$$\vec{v}_{I \in 10} = \vec{h}_{10} \wedge \vec{OI} = e \dot{\theta} \vec{y}_1 + \dot{\theta} \vec{z} \wedge R \vec{y}_0 = -R \dot{\theta} \vec{z}$$

$$\vec{v}_{I \in 10} \neq 0 !$$



$$\begin{aligned} \vec{v}_{I \in 12} &\neq \vec{0} \\ &= \vec{v}_{I \in 10} - \vec{v}_{I \in 20} \\ &= e \dot{\theta} \vec{y}_1 - R \dot{\theta} \vec{x}_0 - \dot{\gamma} \vec{y}_0 \\ &= - (R \dot{\theta} + e \dot{\theta} \sin \theta) \vec{x}_0 + (e \dot{\theta} \cos \theta - \dot{\gamma}) \vec{y}_0 \end{aligned}$$

$$\dot{\gamma} = e \dot{\theta} \cos \theta \Rightarrow \gamma = e \sin \omega t \quad \dot{\theta} = \omega$$

$$\vec{v}_{gr} = - \dot{\theta} (R + e \sin \theta) \vec{x}_0 \quad \dot{\gamma} = e \dot{\theta} \cos \theta$$

$$\{C_{20}\} = \begin{cases} \vec{p}_{20} = m_2 \dot{\gamma} \vec{y}_0 \\ \vec{q}_{G \in 20} = \vec{0} \end{cases}$$

$$\{D_{20}\} = \begin{cases} \vec{d}_{20} = m_2 \ddot{\gamma} \vec{y}_0 \\ \vec{\delta}_{G \in 20} = \vec{0} \end{cases}$$

$$\{C_{10}\} = \begin{cases} \vec{p}_{10} = m_1 e \dot{\theta} \vec{y}_1 \\ \vec{q}_{G \in 10} = \vec{h}_{10} \end{cases}$$

$$\{D_{10}\} = \begin{cases} \vec{d}_{10} = m_1 e (-\ddot{\theta} \vec{x}_1 + \ddot{\theta} \vec{y}_1) \\ \vec{\delta}_{C \in 10} = \frac{d}{dt} (\vec{h}_{10} \wedge \vec{h}_{10}) \vec{z} \end{cases}$$

$$C = \int_0^{2\pi} n^2 n dn d\theta = C \dot{\theta} \vec{z}$$

$$[A_{01}][V_{10}] = \begin{bmatrix} \vec{x}_1 + \vec{y}_1 \\ \vec{0} \end{bmatrix} \begin{bmatrix} \vec{0} \\ \vec{z}_0 \end{bmatrix} = 0$$

$$[G_{010}][V_{10}] = [\vec{0}, -k\dot{\theta}\vec{z}_0][\dot{\theta}\vec{z}_0, \vec{V}_{0e110}] = -k\dot{\theta}^2$$

$$[P_1][V_{10}] = [-m_1 g \vec{z}_0, \vec{0}][\vec{0}, \vec{V}_{0e110}] = 0 \quad \text{car } V_{0e110} \text{ est dans le plan}$$

$$2\mathcal{L}_{0e110} = [V_{10}][C_{10}]$$

$$= \begin{bmatrix} \dot{\theta}\vec{z}_0 \\ (e+a)\dot{\theta}\vec{y}_1 \end{bmatrix} \begin{bmatrix} m(e+a)\dot{\theta}\vec{y}_1 \\ \vec{I}_{Gb1} \cdot \vec{\Omega}_{10} \end{bmatrix} = m(e+a)^2 \dot{\theta}^2 + \dot{\theta}\vec{z}_0$$

On calcule $\vec{I}_{Gb1} \cdot \vec{\Omega}_{10}$: $\vec{GM} \Big|_{\substack{x=0 \\ z=0}}^y = 0$

$$D = \int y z dm$$

$$F = \int x y dm$$

$$\begin{bmatrix} \cdot & \cdot & -C \\ \cdot & \cdot & 0 \\ \cdot & \cdot & C \end{bmatrix} \begin{bmatrix} 0_x \\ 0_y \\ \dot{\theta} \end{bmatrix} = \dot{\theta}(-C\vec{x}_1 + C\vec{z}_0)$$

$$\frac{d}{dt} 2\mathcal{L}_{0e110} = \frac{d}{dt} [m(e+a)^2 + c] \dot{\theta}$$

$$\frac{d\mathcal{L}_{0e110}}{dt} = [m(e+a)^2 + c] \dot{\theta} \ddot{\theta}$$

$$P_{111} = -k\dot{\theta}^2 = \frac{d\mathcal{L}_{0e110}}{dt}$$

$$-k\dot{\theta}^2 = [m(e+a)^2 + c] \dot{\theta} \ddot{\theta}$$

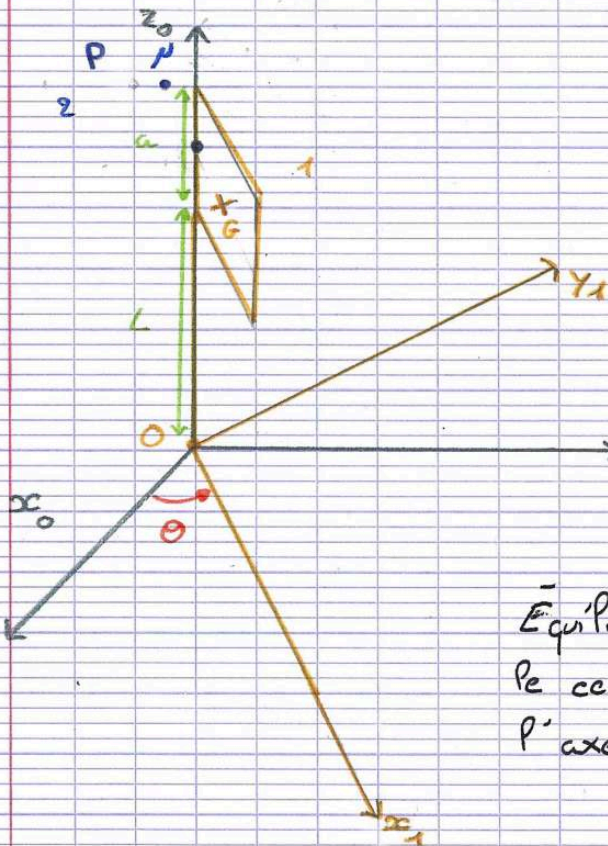
$$\ddot{\alpha} + \lambda \alpha = 0$$

$$\theta = -\frac{\dot{\theta}_0}{\lambda} e^{-\lambda t}$$

$$\alpha = \alpha_0 e^{-\lambda t}$$

$$\dot{\theta} = \dot{\theta}_0 e^{-\lambda t}$$

Girouette équilibre



$$\{V_{10}\} = \begin{cases} \vec{v}_{10} = \dot{\theta} \vec{z} \\ \vec{v}_{O \in 10} = 0 \end{cases}$$

$$\{C_{10}\} = \begin{cases} \vec{p}_{10} = m_1 \cdot \frac{d}{dt} \vec{v}_{10} \wedge \vec{OG} \\ \vec{q}_{O \in 10} = \vec{H}_{O, b_1} \cdot \vec{\Omega}_{10} \end{cases}$$

$$\{D_{10}\} = \begin{cases} m_1 (a/2) (\ddot{\theta} \vec{y}_1 - \dot{\theta}^2 \vec{x}_1) \\ \vec{d}_{O \in 10} = -\epsilon (\ddot{\theta} \vec{x}_1 + \dot{\theta} \dot{\vec{y}}_1) + \dot{\theta} \dot{\vec{z}} \end{cases}$$

Équilibre statique $\Rightarrow$ mettre symétriquement le centre de gravité autour de l'axe de rotation.

$$\{C_{10}\} = \begin{cases} m_1 \dot{\theta} \vec{z} \wedge (a/2 \vec{x}_1 - \vec{z}) = m_1 a/2 \dot{\theta} \vec{y}_1 \\ \begin{bmatrix} | & | & -\epsilon \\ | & | & -D \\ | & | & C \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix} \end{cases} = -\epsilon \dot{\theta} \vec{x}_1 + C \dot{\theta} \vec{z}$$

Théorème du barycentre:

$$m_1 \vec{OG}_1 + m_2 \vec{OG}_2 = (m_1 + m_2) \vec{OG}_{12}$$

$$\mu \vec{OP} + m \vec{OG} = (\mu + m) \vec{OC}$$

$$\mu (z_p \vec{z} + x_p \vec{x}_1) + m \left(\frac{c}{2} \vec{x}_1 + \left(1 + \frac{a}{2}\right) \vec{z} \right) = (\mu + m) z_c \vec{z}$$

$$\mu x_p + m \frac{c}{2} = 0$$

$$\mu z_p + m \left(1 + \frac{a}{2}\right) = (\mu + m) z_c$$

$$\begin{aligned} \vec{v}_{O \in 1210} &= \vec{v}_{O \in 110} + \vec{v}_{O \in 210} \\ \vec{d}_{O \in 1210} &= \vec{d}_{O \in 110} + \vec{d}_{O \in 210} \end{aligned}$$

Exercice de Synthèse : Étude du mouvement d'une bobine

$$\{V_{10}\} = \{\vec{p}_{10} = \dot{\theta} \vec{z}, \vec{q}_{0 \in 10} = \dot{x}_0 \vec{x}\}$$

Au point de contact I entre 1 et 0.

$$\vec{v}_{I \in 10} = \vec{0} = \dot{x}_0 \vec{x} + a \vec{y} \wedge \dot{\theta} \vec{z} = (\dot{x}_0 + a \dot{\theta}) \vec{x}$$

ainsi $\dot{x}_0 = -a \dot{\theta}$

Au point de contact K entre 2 et 1.

$$\begin{aligned} \vec{v}_{K \in 21} = \vec{0} &= \vec{v}_{K \in 210} - \vec{v}_{K \in 10} \\ &= \vec{v}_{A \in 210} - \vec{v}_{K \in 10} \\ &= \dot{x}_A \vec{x} - (\vec{v}_{I \in 10} + \vec{KI} \wedge \vec{p}_{10}) = \dot{x}_A \vec{x} + ((a-b) \vec{y} \wedge \dot{\theta} \vec{z}) \\ &= \dot{x}_A \vec{x} + (a-b) \dot{\theta} \vec{x} \end{aligned}$$

Ainsi $0 = \dot{x}_A - \dot{x}_0 - b \dot{\theta} \Rightarrow \dot{x}_A = \dot{x}_0 + b \dot{\theta}$
 $x_A = x_0 + b \theta$

$$\{C_{10}\} = \begin{cases} \vec{p}_{10} = m \dot{x}_0 \vec{x} \\ \vec{q}_{0 \in 10} = \vec{I}_{O, b_1} \cdot \vec{p}_{10} = \begin{bmatrix} -L \\ -D \\ C \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix} \end{cases}$$

Pour des raisons de symétrie L et D sont nuls.

donc $\{C_{10}\} = \begin{cases} m \dot{x}_0 \vec{x} \\ C \dot{\theta} \vec{z} \end{cases}$

$$\{D_{10}\} = \begin{cases} \vec{d}_{10} = m \ddot{x}_0 \vec{x} \\ \vec{\delta}_{0 \in 10} = C \ddot{\theta} \vec{z} \end{cases}$$

On applique le principe fondamental de la dynamique : $\{A_{T11}\} = \{D_{10}\}$ 3 équations scalaires

On a le théorème de l'énergie cinétique
 $P_{T11}^{ext} = \frac{d}{dt} E_{cT10}$ de plus $2 E_{cT10} = \{v_{10}\} \{C_{10}\}$

Actions à distance: Poids en O $-mg\vec{y}$, $\vec{M}_O P = \vec{OO} \wedge -mg\vec{y}$
 de contact en I $X_{01}\vec{x} + Y_{01}\vec{y}$, $\vec{M}_I P = \vec{OI} \wedge (X_{01}\vec{x} + Y_{01}\vec{y})$
 en K $F\vec{x}$, $\vec{M}_K P = \vec{OK} \wedge F\vec{x}$

En revenant au PFD : $\vec{R}_{T11} = -mg\vec{y} + X_{01}\vec{x} + Y_{01}\vec{y} + F\vec{x}$
 et $\vec{M}_{O \in T11} = \vec{OO} \wedge (-mg\vec{y}) + \vec{OI} \wedge (X_{01}\vec{x} + Y_{01}\vec{y}) + \vec{OK} \wedge F\vec{x}$

On a donc $\begin{cases} \vec{R}_{T11} \\ \vec{M}_{O \in T11} \end{cases} = \begin{cases} \vec{d}_{10} \\ \vec{\delta}_{O \in T10} \end{cases}$

$$\Rightarrow \begin{cases} -mg\vec{y} + X_{01}\vec{x} + Y_{01}\vec{y} + F\vec{x} = m\ddot{x}_0\vec{x} \\ aX_{01}\vec{x} + bF\vec{x} = C\ddot{\theta}\vec{z} \end{cases}$$

$$\text{ie } \begin{cases} X_{01} + F = m\ddot{x}_0 & \vec{x} \in L_1 \\ Y_{01} - mg = 0 & \vec{y} \in L_2 \\ aX_{01} + bF = C\ddot{\theta} & \vec{z} \in L_3 \end{cases}$$

$$\text{Si on fait } L_3 - aL_1 : -aF + bF = -ma\ddot{x}_0 + C\ddot{\theta} \\ F = \frac{C\ddot{\theta} + ma\ddot{x}_0}{b-a}$$

$$\text{En utilisant } \ddot{x}_0 = -a\ddot{\theta} \text{ on a :} \\ F = -\frac{\ddot{\theta}(C+ma^2)}{a-b}$$