Outils Mathématiques – Calcul Différentiel

CD13

1) $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2} \Leftrightarrow \frac{1}{R}=\frac{R_1+R_2}{R_1 R_2} \Leftrightarrow R=\frac{R_1 R_2}{R_1+R_2}$ donc $R=\frac{3300}{9}\ \Omega \approx 113{,}7931034\ \Omega$

2) Calcul direct

$$\Delta R=\left|\frac{\partial R}{\partial R_1}\right|\Delta R_1+\left|\frac{\partial R}{\partial R_2}\right|\Delta R_2$$

$$\frac{\partial R}{\partial R_1}=\frac{R_2(R_1+R_2)-R_1 R_2}{(R_1+R_2)^2}=\frac{R_2^2}{(R_1+R_2)^2}$$

$$\frac{\partial R}{\partial R_2}=\frac{R_1^2}{(R_1+R_2)^2}$$

$$\Rightarrow \Delta R=\frac{R_2^2}{(R_1+R_2)^2}\Delta R_1+\frac{R_1^2}{(R_1+R_2)^2}\Delta R_2$$

$$\Delta R_1=2200\times 0{,}15=330$$
$$\Delta R_2=120\times 0{,}15=18$$

$$\Delta R\approx 17{,}06896\ \Omega$$

Finalement $\Delta R\leqslant 20\ \Omega$

donc $R=(110\pm 20)\ \Omega$

Calcul différentiel

$$\ln R=\ln(R_1 R_2)-\ln(R_1+R_2)=\ln R_1+\ln R_2-\ln(R_1+R_2)$$

donc $d(\ln R)=\frac{dR}{R}=d(\ln R_1)+d(\ln R_2)-d(\ln(R_1+R_2))$

$$\frac{dR}{R}=\frac{dR_1}{R_1}+\frac{dR_2}{R_2}-\frac{d(R_1+R_2)}{R_1+R_2}=\frac{dR_1}{R_1}+\frac{dR_2}{R_2}-\frac{dR_1}{R_1+R_2}-\frac{dR_2}{R_1+R_2}$$

$$\frac{dR}{R}=\left(\frac{1}{R_1}-\frac{1}{R_1+R_2}\right)dR_1+\left(\frac{1}{R_2}-\frac{1}{R_1+R_2}\right)dR_2=\frac{R_2}{R_1(R_1+R_2)}dR_1+\frac{R_1}{R_2(R_1+R_2)}dR_2$$

Ainsi $\frac{\Delta R}{R}=\frac{R_2}{R_1+R_2}\frac{\Delta R_1}{R_1}+\frac{R_1}{R_1+R_2}\frac{\Delta R_2}{R_2}$, et $\frac{\Delta R_1}{R_1}=\frac{\Delta R_2}{R_2}=0{,}15$

Donc $\frac{\Delta R}{R}=0{,}15$, donc ΔR = 17 Ω

CD14

1) $f'=\frac{n'R}{n'-n}=\frac{1{,}53\times 3{,}00}{1{,}53-1{,}00}=8{,}66037\ cm$

On ne peut pas savoir car on ne connaît pas l'incertitude.

2) $df'=\frac{\partial f'}{\partial n'}dn'+\frac{\partial f'}{\partial n}dn+\frac{\partial f'}{\partial R}dR=\left[\frac{R(n'-n)-n'R}{(n'-n)^2}\right]dn'+\left[-\frac{-n'R}{(n'-n)^2}\right]dn+\left[\frac{n'}{n'-n}\right]dR$

$$df'=\frac{-nR}{(n'-n)^2}dn'+\frac{n'R}{(n'-n)^2}dn+\frac{n'}{n'-n}dR$$

3) $\Delta f'=\left|\frac{-nR}{(n'-n)^2}\right|\Delta n'+\left|\frac{n'R}{(n'-n)^2}\right|\Delta n+\left|\frac{n'}{n'-n}\right|\Delta R$

Application numérique : $\Delta f'\approx 0{,}463606\ cm$ → $\Delta f'\leqslant 0{,}5\ cm$

Donc $f'=(8{,}7\pm 0{,}5)\ cm$

CD15

1) t est fixé (ici t = 12 s)

Q ≈ 4,414553.10^{-3} C

$$\Delta Q=\left|\frac{\partial Q}{\partial C}\right|\Delta C+\left|\frac{\partial Q}{\partial E}\right|\Delta E+\left|\frac{\partial Q}{\partial R}\right|\Delta R$$

$$\Delta Q=[E\exp(\frac{-t}{RC})+CE(\frac{-t}{R})(\frac{-1}{C^2})\exp(\frac{-t}{RC})]\Delta C+[Cexp(\frac{-t}{RC})]\Delta E+[CE(\frac{-t}{C})(\frac{-1}{R^2})\exp(\frac{-t}{RC})]\Delta R$$

$$\Delta Q=(E+\frac{Et}{RC})\exp(\frac{-t}{RC})\Delta C+Cexp(\frac{-t}{RC})\Delta E+(\frac{Et}{R^2})\exp(\frac{-t}{RC})\Delta R\approx 1{,}36115\cdot 10^{-3}\,C$$

$$\Delta Q\leqslant 2\cdot 10^{-3}\,C \quad\rightarrow\quad Q=(4\pm 2)\cdot 10^{-3}\,C$$

$$\frac{\Delta Q}{Q}=(1+\frac{t}{RC})\frac{\Delta C}{C}+\frac{\Delta E}{E}+\frac{t}{RC}\frac{\Delta R}{R}\approx 0{,}30833 \quad\rightarrow\quad \frac{\Delta Q}{Q}\approx 31 \quad \%$$

2) $\ln(Q)=\ln(CE\exp(\frac{-t}{RC}))=\ln(C)+\ln(E)-\frac{t}{RC}$

$$d(\ln(Q))=\frac{dQ}{Q}=\frac{dC}{C}+\frac{dE}{E}-d(\frac{t}{RC})=\frac{dC}{C}+\frac{dE}{E}-t\,d(\frac{1}{R}\frac{1}{C})=\frac{dC}{C}+\frac{dE}{E}-t[\frac{1}{R}d(\frac{1}{C})+\frac{1}{C}d(\frac{1}{R})]$$

$$\frac{dQ}{Q}=\frac{dC}{C}+\frac{dE}{E}-t[\frac{1}{R}(\frac{-1}{C^2}dC)+\frac{1}{C}(\frac{-1}{R^2}dR)]=\frac{dC}{C}+\frac{dE}{E}+\frac{t}{RC^2}dC+\frac{t}{CR^2}dR$$

$$\frac{dQ}{Q}=(1+\frac{t}{RC})\frac{dC}{C}+\frac{dE}{E}+\frac{t}{RC}\frac{dR}{R}$$

D'où $\frac{\Delta Q}{Q}=(1+\frac{t}{RC})\frac{\Delta C}{C}+\frac{\Delta E}{E}+\frac{t}{RC}\frac{\Delta R}{R}$

CD16

ATTENTION : convertir les angles en radians (sinon, pb dans la suite)

n ≈ 1,4979114

$$dn=\frac{\partial n}{\partial A}dA+\frac{\partial n}{\partial D}dD=\frac{\frac{1}{2}\cos(\frac{A+D}{2})\sin(\frac{A}{2})-\frac{1}{2}\cos(\frac{A}{2})\sin(\frac{A+D}{2})}{\sin^2(\frac{A}{2})}dA+\frac{\frac{1}{2}\cos(\frac{A+D}{2})}{\sin(\frac{A}{2})}dD$$

$$dn=\frac{1}{2}\frac{\sin(\frac{A}{2}-(\frac{A+D}{2}))}{\sin^2(\frac{A}{2})}dA+\frac{1}{2}\frac{\cos(\frac{A+D}{2})}{\sin(\frac{A}{2})}dD=-\frac{\sin(\frac{D}{2})}{2\sin^2(\frac{A}{2})}dA+\frac{\cos(\frac{A+D}{2})}{2\sin(\frac{A}{2})}dD$$

Ainsi, on a : $\Delta n=\left|\frac{-\sin(\frac{D}{2})}{2\sin^2(\frac{A}{2})}\right|\Delta A+\left|\frac{\cos(\frac{A+D}{2})}{2\sin(\frac{A}{2})}\right|\Delta D$

Donc Δn = 0,015 → Δn ≤ 0,02, donc $n=(1{,}50\pm 0{,}02)$