Outils Mathématiques – Champ scalaires et vectoriels

CSV8

$$\overrightarrow{rot}\,\vec{B} = \vec{\nabla}\wedge\vec{B} = (0-0)\vec{e}_x+(0-0)\vec{e}_y+[2x(x+y)+x^2+y^2-(2y(x-y)-(x^2+y^2))]\vec{e}_z$$
$$= [3x^2+2xy+y^2-2xy+3y^2+x^2]\vec{e}_z$$
$$= (4x^2+4y^2)\vec{e}_z \qquad \neq\vec{0}$$

CSV9

1) $\overrightarrow{rot}\,\vec{C} = \vec{0} \quad \forall M \quad \Rightarrow \quad \exists\ f\ /\ \overrightarrow{grad} f = \vec{C}$

2)

$$\overrightarrow{rot}\,\vec{C} = \vec{0} \quad\Rightarrow\quad (\frac{\partial H}{\partial y}-y)\vec{e}_x+(2x-\frac{\partial H}{\partial x})\vec{e}_y+(0-0)\vec{e}_z$$

$$\begin{cases} \frac{\partial H}{\partial y}-y = 0 \\ 2x-\frac{\partial H}{\partial x} = 0 \end{cases} \Rightarrow \begin{cases} \frac{\partial H}{\partial y} = y & (1) \\ \frac{\partial H}{\partial x} = 2x & (2) \end{cases}$$

D'après (1), $H(x,y) = \frac{y^2}{2}+g(x)$

$\Rightarrow \frac{\partial H}{\partial x} = \frac{dg}{dx} = 2x$, d'après (2). Donc $g(x) = x^2+cte \Rightarrow H(x,y) = \frac{y^2}{2}+x^2+cte$

Or $\vec{C}(0) = \vec{0} \Rightarrow cte = 0$. Donc $H(x,y) = \frac{y^2}{2}+x^2$.

3)

$$\overrightarrow{grad} f = \vec{C} \iff \begin{cases} 2xz = \frac{\partial f}{\partial x} & (1) \\ yz = \frac{\partial f}{\partial x} & (2) \\ \frac{y^2}{2}+x^2 = \frac{\partial f}{\partial z} & (3) \end{cases}$$

D'après (1), $f(x,y,z) = x^2 z+g(y,z)$

$\Rightarrow \frac{\partial f}{\partial y} = \frac{\partial g}{\partial y} = yz$, d'après (2), $\Rightarrow$ $g(y,z) = \frac{y^2}{2}z+h(z)$

Donc $f(x,y,z) = x^2 z+\frac{y^2}{2}z+h(z)$

$\Rightarrow \frac{\partial f}{\partial z} = x^2+\frac{y^2}{2}+\frac{dh}{dz} = x^2+\frac{y^2}{2}$, d'après (3), $\Rightarrow$ $h(z) = cte$

Donc $f(x,y,z) = x^2 z+\frac{y^2}{2}z+cte$

Or f (x,y,z) = 0 dans le plan x0y, c'est-à-dire si z = 0 ⇒ cte = 0

Donc $f(x,y,z) = (x^2+\frac{y^2}{2})z$

CSV11

1) $\operatorname{div}\vec{A} = \frac{\partial A_x}{\partial x}+\frac{\partial A_y}{\partial y} = 1+1 = 2$

$\overrightarrow{rot}\,\vec{A} = (0-0)\vec{e}_z = \vec{0}$

2) $\text{div}\,\vec{B} = \frac{\partial(-y)}{\partial x}+\frac{\partial x}{\partial y} = 0$

$\overrightarrow{rot}\,\vec{B} = 2\vec{e}_z$

CSV12

1) $\text{div}\,\vec{r} = \frac{\partial x}{\partial x}+\frac{\partial y}{\partial y}+\frac{\partial z}{\partial z} = 3$

$\text{div}\,\vec{u} = \text{div}(\vec{e}_r) = \text{div}(\frac{\vec{r}}{r})$

$\frac{\vec{r}}{r} = \frac{x}{\sqrt{x^2+y^2+z^2}}\ \vec{e}_x+\frac{y}{\sqrt{\quad}}\ \vec{e}_y+\frac{z}{\sqrt{\quad}}\ \vec{e}_z$

$= x(x^2+y^2+z^2)^{-1/2}\ \vec{e}_x+y(\quad)\ \vec{e}_y+z(\quad)\ \vec{e}_z$

Méthode directe

$\text{div}(\frac{\vec{r}}{r}) = (x^2+y^2+z^2)^{-1/2}+x(-\frac{1}{2})(2x)(x^2+y^2+z^2)^{-3/2}+(\quad)\,en\,y+(\quad)\,en\,z$

$= (\ )^{-1/2}-x^2(\ )^{-3/2}+(\ )^{-1/2}-y^2(\ )^{-3/2}+(\ )^{-1/2}-z^2(\ )^{-3/2}$

$= \frac{3}{\sqrt{x^2+y^2+z^2}}-\frac{x^2+y^2+z^2}{(x^2+y^2+z^2)^{-3/2}} = \frac{3}{\sqrt{\quad}}-\frac{1}{\sqrt{\quad}}$

$= \frac{2}{\sqrt{x^2+y^2+z^2}}$

Méthode avec relat° du CSV10 : $\text{div}(G\vec{A}) = \vec{A}.\overrightarrow{grad}\,G+G\,\text{div}\,\vec{A}$

$\text{div}(\frac{\vec{r}}{r}) = \vec{r}.\overrightarrow{grad}\frac{1}{r}+\frac{1}{r}\text{div}\,\vec{r} = \vec{r}.[\frac{\partial}{\partial r}(\frac{1}{r})\vec{e}_r]+\frac{1}{r}3 = -\frac{1}{r^2}\vec{e}_r.\vec{r}+\frac{3}{r} = -\frac{1}{r^2}\frac{\vec{r}}{r}.\vec{r}+\frac{3}{r} = \frac{3}{r}-\frac{1}{r}$

$\text{div}(\frac{\vec{r}}{r}) = \frac{2}{r} = \frac{2}{\sqrt{x^2+y^2+z^2}}$

Notes :
cte : constante(s)
° : -ion(s)